Harmonic Distortion in a Class of Linear Active Filter Networks

PETER J. BILLAM

Filtek A.G., Biel, Switzerland

Journal of the Audio Engineering Society, June 1978, Volume 26 Number 6, pp.426-429

Due to the positive feedback inherent in most active filter networks, any harmonic distortion in the amplifier is increased. This worsening is calculated analytically for a certain class of networks, and a simple but accurate breadboard method for its measurement is suggested.

INTRODUCTION: This paper considers the behaviour of the circuit shown in Fig. 1. The amplifier has gain g, a high input-impedance, and a low output-impedance. Any input conductance can be regarded as part of the linear network. We are going to postulate that this amplifier distorts, with harmonic distortion coefficients $k_2, k_3, ...$, following normal audio terminology. This means that if we feed a 1-volt sine wave at 1 kHz into the amplifier, the output will be a 1-kHz sine wave of amplitude g volts, plus a 2-kHz sine wave of amplitude $g.k_2$ volts, a 3-kHz sine wave of amplitude $g.k_3$ volts, etc.

We will also assume that the k are sufficiently small (say, less than 1%) that their second-order products may be neglected.

The linear network is made up of linear resistors, capacitors and inductors, and is assumed to obey the following restrictions:

1) The input impedance at both inputs is non-zero.
2) The output impedance is non-infinite.
3) The resulting circuit of Fig. 1 is stable.

If the third of these conditions is not met, we have a high-distortion oscillator; if it is obeyed, we have a filter. Many useful filters are of this class, for example Sallen and Key filters [1], bootstrapped notch filters [2], etc.

We will show that the amplitudes of the distortion components at the output are worse than the amplifier's own values by a factor, the distortion-aggravation-factor, equal to the gain of the positive feedback loop consisting of the amplifier and the β input of the network. This gain is known from feedback theory to be $1/(1 - g\beta)$.

The detailed analysis is presented in Section 3.

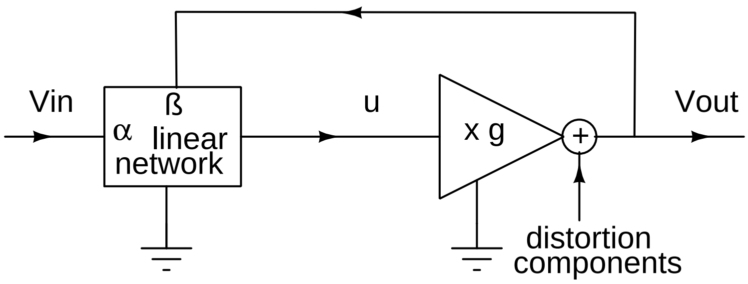

Figure 1: Circuit under consideration

1 CHARACTERISATION OF THE AMPLIFIER

The amplifier is assumed to be slightly nonlinear, and its distortion is characterised as follows, following conventional audio terminology. For an input of $Ae^{j\omega t}$, the output is

$$gAe^{j\omega t} + \sum_{j=1}^{\infty} gA \cdot k_n \cdot e^{nj\omega t} \tag{1}$$

where g is the gain of the amplifier and k_n is the nth-harmonic distortion coefficient.

The total harmonic distortion (THD) is then given by

$$THD = \sqrt{\sum_{j=2}^{\infty} |k_n|^2}$$

In general, k_n is a phasor function of ω and of gA. Second-order products of the k_n will be neglected.

2 CHARACTERISATION OF THE LINEAR NETWORK

The linear network, which is being driven from two low-impedance sources, is characterised by the transfer functions corresponding to the two inputs, that is, $\alpha(\omega)$ for the α input and $\beta(\omega)$ for the β input.

3 ANALYSIS

The signal v_{in} is assumed to be a pure sine wave,

$$v_{in} = V_{in} \cdot e^{j\omega t} \tag{2}$$

The signals u and v_{out} are expanded as follows:

$$u = \sum_{1}^{\infty} U_n \cdot e^{nj\omega t} \tag{3}$$

$$v_{out} = V_{out} \cdot e^{j\omega t} + \sum_{2}^{\infty} V_{out} \cdot k_n^{'} \cdot e^{nj\omega t} \tag{4}$$

where the coefficients $k_n^{'}$ are the resultant harmonic distortion coefficients of the whole circuit.

Thus neglecting second-order products of k and $k^{'}$, the amplifier produces an output signal of

$$v_{out} = \sum_{1}^{\infty} gU_n \cdot e^{nj\omega t} + \sum_{2}^{\infty} gU_1 \cdot k_n^{'} \cdot e^{nj\omega t} \tag{5}$$

where the first term represents the amplification of a distorted input, and the second represents the new distortion of the fundamental.

Then the linear network produces an output signal of

$$u = \sum_{1}^{\infty} U_n \cdot e^{nj\omega t} = \alpha\omega \cdot V_{in} \cdot e^{j\omega t} + \sum_{1}^{\infty} g\beta(n\omega) \cdot U_n \cdot e^{nj\omega t} + \sum_{2}^{\infty} g\beta(n\omega) \cdot U_1 \cdot k_n^{'} \cdot e^{nj\omega t} \tag{6}$$

Equating terms on both sides of Eq. (6),

$$U_1 = \alpha(\omega) \cdot V_{in} + g\beta(\omega) \cdot U_1 \tag{7}$$

and for $n \neq 1$

$$U_n = g\beta(n\omega) \cdot U_n + gU_1 \cdot \beta(n\omega) \cdot k_n \tag{8}$$

Then from Eq.(8), for $n \neq 1$

$$U_n = \frac{gU_1 \cdot \beta(n\omega) \cdot k_n}{1 - g\beta(n\omega)} \tag{9}$$

and from Eq.(7),

$$U_1 = \frac{\alpha(\omega) \cdot V_{in}}{1 - g\beta(\omega)} \tag{10}$$

Substituting Eqs. (9) and (10) into Eq.(5) to find v_{out} gives

$$v_{out} = gU_1 \cdot e^{j\omega t} = \sum_2^\infty \frac{g^2 U_1 \cdot \beta(n\omega) \cdot k_n \cdot e^{nj\omega t}}{1 - g\beta(n\omega)} + \sum_2^\infty gU_1 \cdot k_n \cdot e^{nj\omega t} \tag{11}$$

But Eq.(4) is

$$v_{out} = V_{out} \cdot e^{j\omega t} + \sum_2^\infty V_{out} \cdot k_n^{'} \cdot e^{nj\omega t}$$

Thus equating terms between Eqs.(4) and (11),

$$V_{out} = gU_1 \tag{12}$$

and

$$V_{out} \cdot k_n^{'} = \frac{g^2 U_1 \cdot \beta(n\omega) \cdot k_n}{1 - g\beta(n\omega)} + gU_1 \cdot k_n \tag{13}$$

Substituting Eq.(10) into Eq.(12), we get

$$V_{out} = \frac{g\alpha(\omega)}{1 - g\beta(n\omega)} \cdot V_{in} \tag{14}$$

which is the transfer function of the circuit, neglecting distortion.
Substituting Eq.(12) into Eq.(13) we get

$$V_{out} \cdot k_n^{'} = \frac{V_{out} \cdot g\beta(n\omega) \cdot k_n}{1 - b\beta(n\omega)} + V_{out} \cdot k_n \tag{15}$$

or

$$k_n^{'} = k_n \cdot \frac{1}{1 - g\beta(n\omega)} \tag{16}$$

Note that V_{out} has not disappeared, since k_n is in general a function of the output amplitude.

4 COMMENT

This equation means that the fact of employing the amplifier in this filter circuit worsens its distortion coefficients by the factor

$$1/(1 - g\beta(n\omega))$$

which we have called the distortion aggravation factor.

Note that in the circuits of Fig. 2, and indeed in almost all practical filters of this class, $\beta(0) = 0$ and $\beta(\infty) = 0$; thus at frequencies well above and below the characteristic frequencies of the filter, the amplifier distortion is not aggravated by the action of the filter.

It may also be pointed out that a similar analysis to that above can be applied to any foreign signal introduced by the amplifier, such as noise, for example. The amplifier's spot noise at the frequency ω will also be worsened by $1/(1 - g\beta(n\omega))$.

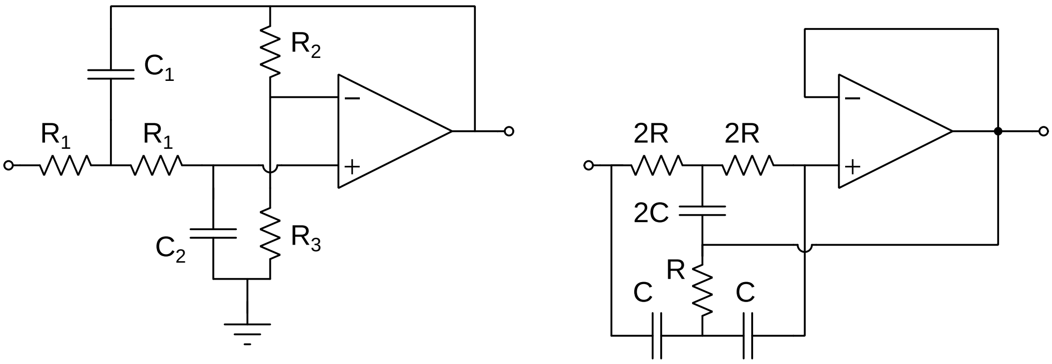

Figure 2: a: Sallen and Key filter b: Bootstrapped notch filter

5 METHOD OF MEASUREMENT

The estimation of the distortion-aggravation factor is of some interest to the designer since it determines the quality of the amplifier he is obliged to use.

The following breadboard method of measurement is proposed.

5.1 Step 1 (Fig.3)

When V_{in} is a sine wave of frequency $n\omega$, V_{out} is measured. By Eq. [14] this will be

$$V_{out}{}^{1} = \frac{g\alpha(n\omega)}{1 - g\beta(n\omega)} \cdot V_{in}$$

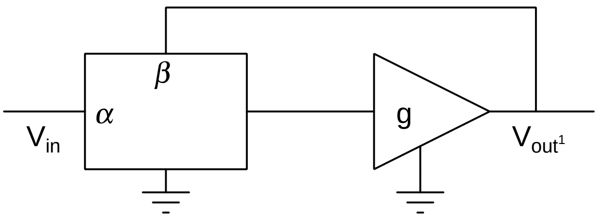

Figure 3: Step 1

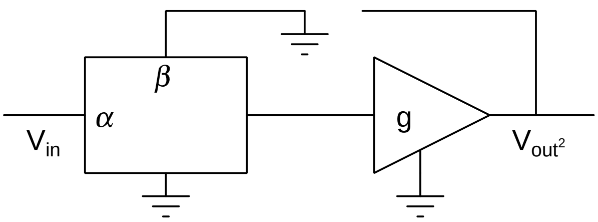

Figure 4: Step 2

5.2 Step 2 (Fig.4)

Now the loop is broken, and the β input of the linear network is connected to earth (Fig.4), and the output voltage is measured again. Now

$$V_{out}{}^{2} = g\alpha(n\omega) \cdot V_{in} \tag{17}$$

by the definition of α. Thus the ratio of the two measurements is

$$\frac{{V_{out}}^1}{{V_{out}}^2} = \frac{1}{1 - g\beta(n\omega)} \tag{18}$$

which is the distortion-aggravation factor from Eq. (16).

This method is quick and precise and applies to all filters of this class: Bessel, Butterworth, Tschebyscheff, or indeterminate, and for engineers with limited access to the necessary hardware or software [3], [4] it forms a practical alternative to a computer analysis of the linear network.

6 EXAMPLES

1) As an illustration, the third-order low-pass Tschebyscheff filter [5] was constructed (Fig.5) which has a predicted corner frequency of 1940 Hz. The measurements are shown in Table 1.

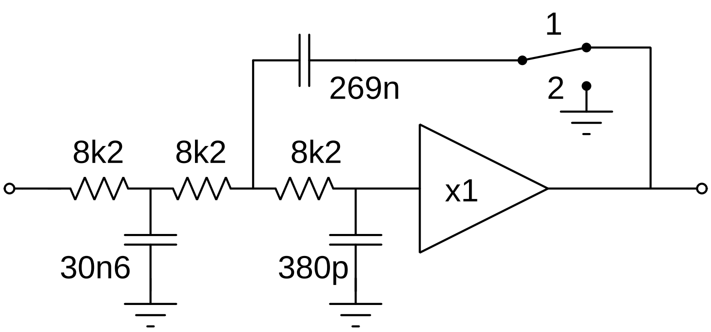

Figure 5: Third-order Tschebyscheff filter

It will be observed that over a range of more than three octaves the distortion-aggravation factor of this filter is greater than 10, and that near the corner frequency it rises to more than 80. With this information, the designer would, for example, be able to rule out of hand the use of a high-distortion amplifier (such as a simple emitter follower with its varying V_{be} distortion, or a μA741 with its varying input-stage, low open-loop gain at high frequencies, and high slew-induced distortion [6]) to realise amplifier A.

$n\omega/2\pi$ (Hz)	V_1 (volts)	V_2 (volts)	Distortion Aggravation Factor V_1 / V_2
330	6.70	0.723	9.3
390	6.53	0.598	10.9
470	6.28	0.486	12.9
560	6.04	0.396	15.2
680	5.86	0.316	18.5
820	5.60	0.256	21.9
1000	5.60	0.195	28.7
1200	5.80	0.148	39.2
1500	6.63	0.106	62.6
1800	6.53	0.0806	81.0
2200	3.15	0.0568	55.5
2700	1.28	0.0384	33.3
3300	0.606	0.0626	9.68

It is even possible that the reputation possessed by Tschebyscheff filters in audio circles for "sounding too harsh" is due not to the sharp rolloff, but to the distortion-aggravation effect discussed in this paper.

2) Dobkin discusses [2] the 60-kHz notch filter of bootstrapped twin-T form (Fig.6). The factor $1/(g\beta(nw)$ can be read directly from his Figure 2; at 51 Hz and at 71 Hz it is approximately 30 dB $\simeq$ 30 times; at 42 and 83 Hz it is approxiamately 20 dB = 10 times.

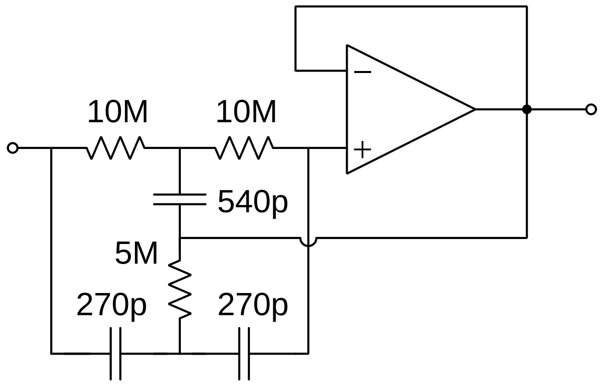

Figure 6: High-Q notch filter

Thus here too a certain prudence would be indicated in the choice of amplifier, particularly if the notch was being constructed at a higher frequency, such as 6 kHz, where distortion components are more common.

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THE AUTHOR

Peter J. Billam received a B.SC. degree in physics at Imperial College, London, in 1969, and after one year's research in computational plasma simulation, played for three years as a professional musician. He is now working in Switzerland designing audio and control equipment for broadcast applications.

Mr. Billam is an associate member of the Institution of Electrical Engineers.