Distortion in Positive- and Negative-Feedback Filters

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It is known that the harmonic distortion of an active filter is greater than the distortion of the operational amplifier itself. Positive- and negative-feedback linear filters, namely, Butterworth, Chebyschev, Bessel and band-pass filters, are analysed. The distortion multiplication factor K_d is defined and plotted versus frequency. The maximum values of K_d found for several filter configurations by means of a computer program are given in a form useful for filter designers.

INTRODUCTION: The fact that the harmonic distortion in a positive-feedback (PF) active filter is much greater than the distortion in its operational amplifier is not new [1]. The closer the distortion is to the filter cutoff-frequency, the larger is its increase. Distortion can be increased by up to two orders of magnitude. Consequently it has been suggested that the unpleasant auditive sensation caused by some filters is due to this fact, and not to the excessive phase rotation usually associated with high-slope filters.

This conclusion was applied to negative-feedback (NF) active filters in a previous publication [2], but the results of this work are not easily applicable to the improvement of filter design, since they involve the solving of complex equations.

Our investigation has a two-fold goal. First we want to find simple equations which will enable the designer to quickly estimate the increase of distortion in each active filter. Second, we want to try to make the method as widely applicable as possible, so that it holds for filters not included in this paper as well as for filters to be developed in the future.

1 DISTORTION MULTIPLICATION FACTOR

The definition of a distortion multiplication factor will help us to estimate quickly the particular behaviour of each filter. We will call it K_d and it will be expressed in decibels for ease of notation and calculus.

$$K_d(f) = 20\ \log\ \frac{\text{distortion of active filter at frequency } f}{\text{distortion of operational amplifier with } g = 1} \tag{1}$$

It is evident that $K_d(f)$ will depend on the frequency (See Fig. 13). In this work, as will be explained later, it is demonstrated that the behaviour of $K_d(f)$ is represented by a bell-shaped curve, which has its maximum value near the filter cutoff-frequency f_3. This holds not only for low-pass or high-pass filters, but also for band-pass filters at the center-frequency f_0.

Then we can concentrate only of the maximum value of $K_d(f)$. Thus we are transforming a problem of complex solution into a much simpler problem, which consists of determining the maximum distortion to be obtained with each filter configuration. We know beforehand that for all filters this value will be near f_3 or f_0. Then

$$K_d = [K_d(f)]_{max} \tag{2}$$

We have thus developed a method that can be applied easily to filter design. For example, we want to design a filter with a harmonic distortion less than $0.1\% = -60$ dB. Looking at the charts of K_d for each configuration, we see that, for example, $K_d = 25$ dB. Consequently, the operational amplifier will have a distortion with unity gain below $-(60+25) = -85$ dB (0.0056%).

2 K_d IN POSITIVE- AND NEGATIVE-FEEDBACK FILTERS

The operational amplifier with which the filter is going to be built is discussed next. When the operational amplifier is connected for unitary gain and is driven by an ideal frequency generator V_1, it delivers to its output

$$V_0 \;=\; V_1 + V_2 + V_3 + \cdots + V_n$$

where V_1 is the fundamental frequency and V_2, $V_3, \cdots$ are harmonic distortion components.

The harmonic components will be multiplied by K_d, thus increasing the distortion measured at the filter output. The equivalent circuit of the operational amplifier with distortion shown in Fig. 1(a) will be used to anlyse this problem. The distortion generators V_n are in series with an ideal distortionless amplifier. However, this circuit can be simplified even further if we take into account that the distortion values of an operational amplifier are generally low. Consequently the second-order products will be negligible, and it will not be necessary to analyse all the generators working at the same time. We will just have to

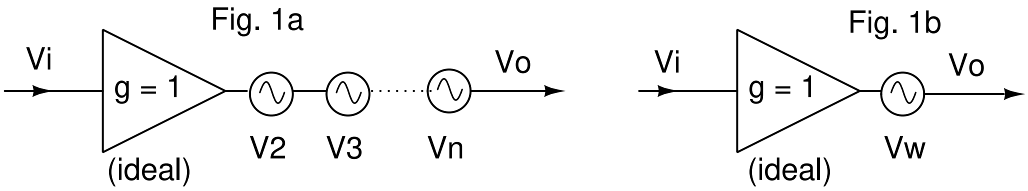


Figure 1: Equivalent circuit of operational amplifier with distortion

replace $V_1 + V_2 + V_3 + \cdots + V_n$ by a single distortion generator V_ω whose frequency can be changed so that it replaces any of the harmonics [Fig. 1(b)]. From the viewpoint of circuit theory this implies assuming that the system is linear, and in fact it almost is, since the second-order nonlinearity is negligible.

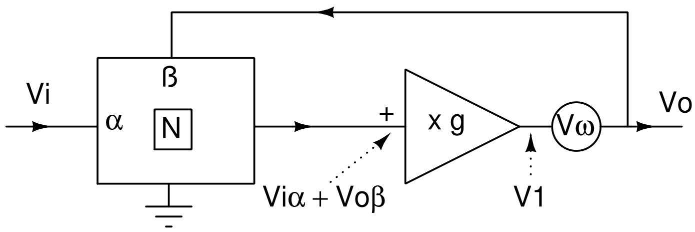


Figure 2: Positive-feedback active filter

Fig. 2 shows the equivalent circuit of a positive-feedback active filter. This circuit is valid for any type of filter. The passive network N is defined by two transfer factors α and β. Then for this circuit, and for $V_i = 0$,

$$\begin{aligned} V_1 &= g\beta(\omega)V_0 \\ V_0 &= V_\omega + V_1 \;=\; V_\omega + g\beta(\omega)V_0 \end{aligned}$$

$$V_0 = \frac{V_\omega}{1 - g\beta(\omega)V_0}$$

Since $$K_d(f) = 20 \log \frac{V_0}{V_\omega}$$ then for a positive-feedback filter

$$K_d(f) = 20 \log \frac{1}{1 - g\beta(\omega)} \tag{3}$$

Equation 3 will enable us to calculate K_d according to the passive network transfer $\beta(\omega)$. A maximum value of $K_d(f)$ will be obtained when $\beta(\omega)$ is real and has a maximum (that is, when it is close to 1).

Let us now analyse the negative-feedback filter Fig. 3.

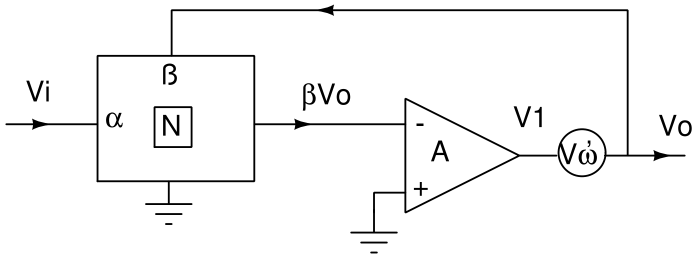


Figure 3: Negative-feedback active filter

$$\begin{aligned} V_1 &= -A\,\beta(\omega)\,V_0 \\ V_0 &= V_1 + V_\omega^{'} \;=\; V_\omega^{'} - A\,\beta(\omega)V_0 \\ V_0 &= \frac{V_\omega^{'}}{1 + A\,\beta(\omega)V_0} \end{aligned}$$

If we apply the definition of Eqn. 1,

$$K_d\,(f) = 20 \log \frac{[V_o]_\omega^\beta}{[V_o]_\omega^{\beta=1}}$$

that is, we consider $\beta = 1$ in order to obtain in the denominator the distortion corresponding to the closed-loop condition ($g = 1$) given by $V_\omega^{'}/(1 + A)$. From this viewpoint, and if we want to be accurate, we must say that $V_\omega^{'}$ is not the open-loop distortion, but the equivalent input-distortion as defind by Baxandall [3]. Finally the quotient will be

$$\begin{aligned} K_d\,(f) &= 20 \log \frac{V_\omega^{'}\,/\,[1 + A\beta(\omega)]}{V_\omega^{'}\,/\,(1 + A)} \\ &= 20 \log \frac{1 + A}{1 + A\beta(\omega)} \end{aligned}$$

Taking into account that $A \gg 1$, we have, for negative-feedback filters,

$$K_d\,(f) = 20 \log \frac{1}{\beta(\omega)} \tag{4}$$

Equations (3) and (4) will enable us to calculate K_d for all active filters by just solving the passive network in order to obtain β. Eqs. (3) and (4) refer the filter distortion to the value that corresponds to the amplifier with unitary gain. However, in real operational amplifiers, distortion can have different values depending on which input is selected, positive or negative. This is due to the inherent distortion of the input differential pair, which is, respectively, included in or excluded from the feedback loop. This discrepancy with the theoretical model can generally be neglected and does not alter noticeably the results obtained from Eqs. (3) and (4).

3 K_d VALUES FOR ORDINARY FILTERS

Since an analytic solution of the passive network beta transfer function is quite complex, we decided to use a computer program, the CNAP, supplied by Hewlett-Packard. Thus the charts for Figs. 4 and 6 were obtained. Th program was also used to determine whether the maximum value of $K_d(f)$ occurred exactly at the cutoff-frequency. With this important information we went on to solve the network analytically, but just for the K_d frequency. Likewise we were able to obtain simple equations to calculate K_d values that will enable the designer to make do without charts or computer programs. In Figs. 4 and 6 the computed exact K_d values are also included.

Fig. 4 shows the popular third-order Chebyschev filter. For a filter with a 2-dB ripple the maximum $K_d(f)$ value is 38.4 dB, which coincides with the value obtained by Billam [1] by means of experimental methods. It must be noted that this filter multiplies 83 times the distortion of the amplifier used.

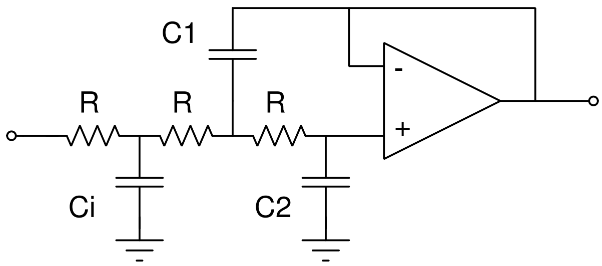


Figure 4: Third-order positive-feedback Chebyschev filter

Ripple-factor ϵ [dB]	K_d [dB]
0.5	28.4
1	32.4
2	38.4
3	43.4

Fig. 5 shows, as a calculation example, the five-node network used to solve the filter in Fig. 4. The generator placed between nodes 4 and 5 has the function of subtracting 1 V from the voltage in node 4. The output obtained is thus $\beta(\omega) - 1$. This output enables the computer to print the values of $20 \log[\beta(\omega) - 1]$, which will be the same as the $K_d(f)$ value given by Equ. 3, except for the sign. As regards negative-feedback filters, it is not necessary to use the generator since the β transfer can be obtained directly.

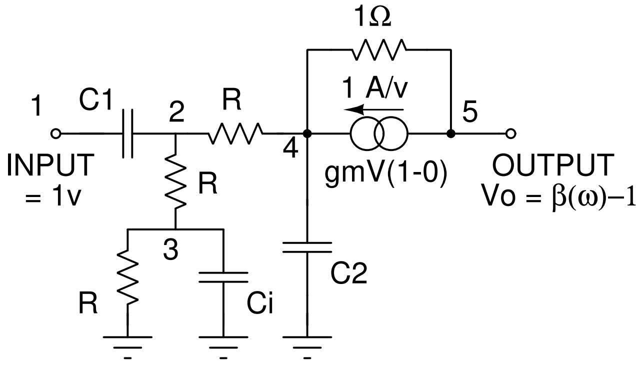
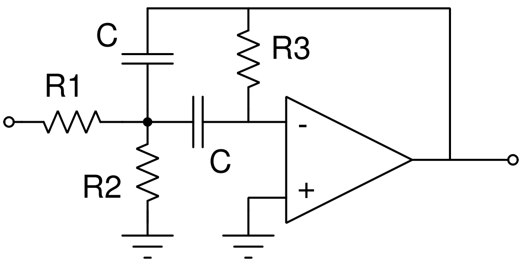


Figure 5: Network of third-order Chebyschev filter, used for CNAP computer program



$$R_1 = \frac{1}{C}\,\frac{1}{\Delta\omega}$$

$$R_2 = \frac{1}{C}\,\frac{\Delta\omega}{2\omega_0^2 - \Delta\omega^2}$$

$$R_3 = \frac{1}{C}\,\frac{2}{\Delta\omega}$$

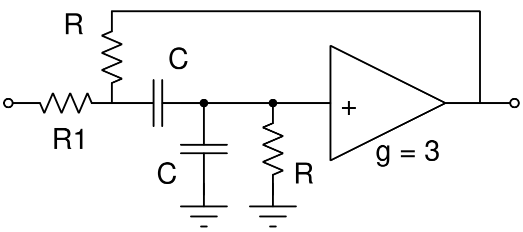
$$Q = \frac{\omega_0}{\Delta\omega}$$

Figure 6: Negative-feedback bandpass filter

Exact computed values:

Q	K_d [dB]
1	9.5
3	25.6
6	37.3
10	46.1
20	58.1

Exact expression: $20\,\log\left(1 + 2Q^2\right)$



$$R_1 = \frac{2}{C}\,\frac{1}{\Delta\omega}$$

$$R \;= \frac{1 + \sqrt{1 + 16Q^2}}{4\,C\,\omega_0\,Q}$$

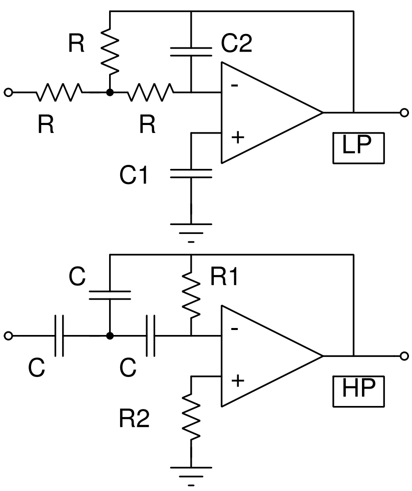
$$Q = \frac{\omega_0}{\Delta\omega}$$

Figure 7: Positive-feedback bandpass filter

Exact computed values:

Q	K_d [dB]
0.5	5.7
1	10.5
3	19.4
10	29.7
40	40.8

Approximate expression: $20\,\log\left(1/2 + 3Q\right)$
(error < 0.8 dB)



$$\omega_0 = \frac{1}{R\,\sqrt{C_1 C_2}}$$

$$Q = \frac{1}{3}\,\sqrt{\frac{C_1}{C_2}}$$

$$Q = \frac{1}{3}\,\sqrt{\frac{R_1}{R_2}}$$

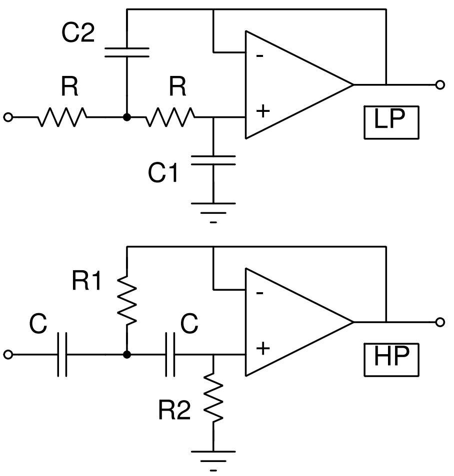
Figure 8: Negative-feedback lowpass and highpass filters

Exact computed values:

Q	K_d [dB]
0.5	5.2
0.707	8.3
1.5	17.9
3	29.0
10	49.6
20	61.6
40	73.5

Approximate expression: $20 \log\left(1 + 3Q^2\right)$
(error < 0.3 dB)

In the charts of Figs. 6 - 9 the resistors and capacitors were standardised in accordance with the Q factors of the filter. This fact is very important, because the designer always knows beforehand the Q for which the filter must be designed. Inversely, if the filter has already been calculated by means of charts or computer programs, the designer will be able to find the Q of each section of the filter, and from it the K_d factor, since the former is dependent on the relationship of capacitors or resistors. For this purpose Figures 6 - 9 include, beside the charts, the equations that enable the designer to calculate the filters and their Q.



$$\omega_0 = \frac{1}{R\sqrt{C_1 C_2}}$$

$$Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$

$$Q = \frac{1}{2}\sqrt{\frac{R_2}{R_1}}$$

Figure 9: Positive-feedback lowpass and highpass filters

Exact computed values:

Q	K_d [dB]
0.5	3.5
0.707	6.0
1.5	14.8
3	25.6
10	46.0
20	58.1
40	70.1

Exact expression: $20 \log\left(1 + 2Q^2\right)$

We shall now see the analytic expression that provides K_d as a function of Q for a low-pass filter with positive feedback (Fig. 9). By solving the network with a computer, we learned that the maximum value for $K_d(f)$ occurs for a frequency identical to the cutoff frequency f_3. Thus we will calculate the β value for that frequency (designated ω_0 in Fig.9).

From the low-pass circuit in Fig.9 we can build the β network of Fig.10. In order to solve the latter, we will take the following into account.

Standardising $\omega_0 = 1$, we have $$R\sqrt{C_1 C_2} = 1$$

Then $$C_2 = \frac{1}{R^2\, C_1} \tag{5}$$

and $$4\,Q^2 = \frac{C_2}{C_1}$$

Hence $$C_2 = 4\,Q^2\,C_1 \tag{6}$$

Equating Eqs. (5) and (6) $$2\,Q\,R\,C_1 = 1 \tag{7}$$

Solving $1/C_1$ and $1/C_2$ from (6) and (7) $$\frac{1}{C_1} = 2QR \quad\text{and}\quad \frac{1}{C_2} = \frac{R}{2Q} \tag{8}$$

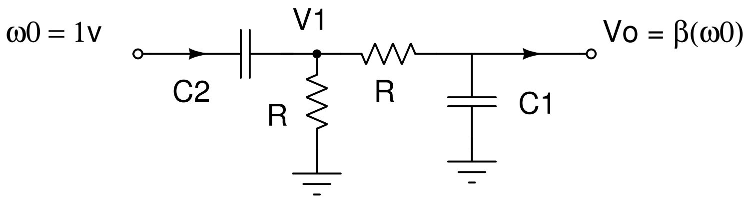


Figure 10: β network for lowpass filter with positive-feedback

We now calculate V_1 (Fig. 10): $$V_1 = \frac{R\,Z_{RC_1}\,/\,(R + Z_{RC_1})}{X_{C_2}\; +\; R\,Z_{RC_1}/(R + Z_{RC_1})}$$

But $$Z_{RC_1} = R\; -\; j\,1/C_1$$

because $\omega_0 = 1$, and so $$V_1 = \frac{R\,(R - j/C_1)}{-j\,/\,C_2(R + R\,(R - j/C_1)) + R\,(R - j/C_1)}$$

Replacing by Eqn(8) and solving: $$V_1 = \frac{Q - j2Q^2}{-j(2Q^2 + 1)} \tag{9}$$

and $$\beta(\omega_0) = V_1\,\frac{X_{C_1}}{R + X_{C_1}} \;=\; V_1\,\frac{-j/C_1}{R - j/C_1}$$
$$= V_1\,\frac{-j2Q}{1 - j2Q}$$

Using V_1 from (9) and solving: $$\beta(\omega_0) = \frac{2Q^2}{2Q^2 + 1}$$

Then $$K_d = 20\,\log\frac{1}{1 - \beta(\omega_0)}$$
$$= 20\,\log\frac{1}{1\; -\; 2Q^2/(2Q^2 + 1)}$$

and hence $$K_d = 20\,\log(2Q^2 + 1) \tag{10}$$

Eqn. (10) allows us to calculate the value of K_d as a function of Q for the low-pass and high-pass filters in Fig. 9.

In our second example we examine a negative feedback low-pass filter. The filter in Fig. 8 has its maximum value of $K_d(f)$ very close to f_3, but for lower values of Q, $K_d(f)$ is slightly displaced. For higher values of Q it is almost coincident with f_3. As the previously-mentioned drift is very small, we use an approximate expression of K_d for design purposes only, Thus we consider that the ω_0 value of Fig. 8 gives us the maximum value of $K_d(f)$. Fig. 11 shows the β network to be analysed.

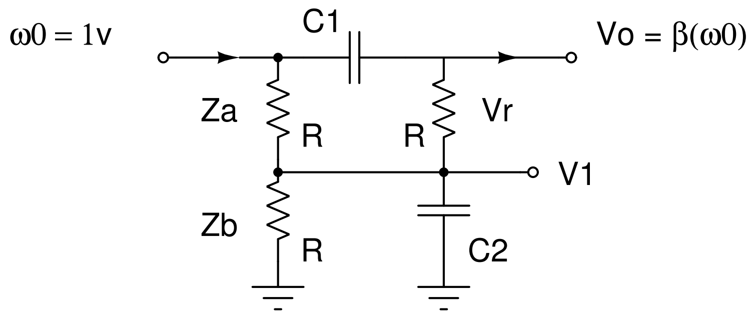


Figure 11: β network for lowpass filter with negative-feedback

Similarly to our previous example, we find

$$\frac{1}{C_1} = 3\,Q\,R \qquad \frac{1}{C_2} = \frac{R}{3\,Q} \tag{11}$$

We calculate Z_A and Z_B and substitute Eqn. (11)

$$Z_A = R\,\frac{1-j3Q}{2-j3Q} \qquad Z_B = R\,\frac{1-j3Q}{2-j3Q}$$

So $$V_1 = \frac{Z_B}{Z_A + Z_B}$$

Replacing and solving $$V_1 = \frac{2-3jQ}{3+9Q^2-j3Q}$$

also $$\beta(\omega_0) = V_1 + V_R$$

and $$V_R = (1-V_1)\,\frac{R}{R-j/C_1}$$

Replacing and solving further, $$\beta(\omega_0) = \frac{3}{3+9Q^2-j3Q}$$

then
$$\begin{aligned} K_d &= 20\,\log\left|\frac{1}{\beta(\omega_0)}\right| \\ &= 20\,\log\,[\,(1+3Q^2)^2 + Q^2\,]^{1/2} \end{aligned}$$

and hence approximately $$K_d = 20\,\log\,(\,1+3Q^2\,) \tag{12}$$

The value given by Eqn. (12) is only approximate. However, if we compare it with the exact values of the chart in Fig. 8, we see that its maximum error is 0.3 dB. Consequently, it can be considered accurate for practical purposes. It is possible to find analytic expressions for the remaining types of active filters following the above procedures.

In Figs. 6-9 the designer will find the exact or an approximate expression that will permit calculating K_d directly. Note that among the the positive- and negative-feedback realisations of low-pass or high-pass filters (Figs. 8 and 9) the differences in K_d values for the same Q are in fact small. Conversely, in second-order pass-band filters the differences are very important, since the K_d values for the K_d value for the positive-feedback configuration is much lower than that for the negative-feedback configuration.

It is obvious that we have not dealt with all the types of active filters. However, the method we have described holds for any kind of filter, either by the solution of the β network by computer, or by analytical calculus for frequencies f_3 or f_0.

4 HOW TO CALCULATE Q_{max} FOR BUTTERWORTH, CHEBYSCHEV AND BESSEL FILTERS

When we design a filter of higher than second order, we build it with two or more second-order sections. Each of these sections will have its own distortion due to the Q with which it operates.

As the distortion increases by K_d and since K_d is proportional to Q^2, the overall filter distortion will be almost the same as the distortion in the section of higher Q. This is why the designer should find the Q that corresponds to each pair of conjugated poles. The maximum value (corresponding to a specific filter section) will then give the overall distortion. The value of Q for a pair of poles $\alpha \pm j\beta$ is given by

$$Q = \frac{\sqrt{\alpha^2 + \beta^2}}{2\,\alpha} \tag{13}$$

As regards the Butterworth polynomials, the value can be calculated easily, since the poles are given by Weinberg [4].

$$S_{2V+1} = -\sin\,\frac{(2V+1)\pi}{2\,n} + j\,\cos\,\frac{(2V+1)\pi}{2\,n}$$

where $V = 0, 1, 2, ..., n-1$, with n being the filter order.

If we now apply Eqn. (13), $$Q = \frac{1}{2\sin[\,(2V+1)\pi/2n\,]}$$

For $V = 0$ we have $$Q_{max} = \frac{1}{2\sin(\pi/2n)} \tag{14}$$

Then we can find the Q_{max} for an n-th order Butterworth filter by means of Eqn. (14). For a simple, second-order filter, $Q = 0.707$, and for $n = 10$, $Q_{max} = 3.196$. If we use a positive-feedback configuration similar to the one in Fig. 9, then $K_d = 6$ dB in the first example and 26.6 dB in the second. This implies that if the same distortion is desired in both filters, then in the second example ($n = 10$) it is necessary to have an amplifier with ten times less distortion than in the first example.

For Chebyschev filters the analytic expression is more complex. So it will be convenient to obtain the root values from a polynomial chart [4]. To have a comparative reference, with a ripple of 1 dB and $n = 4$, we have $Q_{max} = 3.56$. On the other end, with a ripple of 3 dB and $n = 10$, $Q_{max} = 38.85$. In this case, Eqn. (13) is used to calculate the Q values.

Bessel filters can also be calculated according to the roots found in the charts, but the Q values are so low that the do not cause any noticable increase in distortion. For instance, for $n = 10$, we have $Q_{max} = 1.42$ in a Bessel filter.

Eqn. (13) will be useful to calculate any other type of filter, provided the roots of the approximate polynomial are known.

5 EXPERIMENTAL MEASUREMENT of $K_d(f)$

The following method can be used if we want to design a new kind of active filter with K_d unknown. Instead of calculating the K_d value, it is sometimes faster to measure it directly. For this purpose the simple circuit layouts in Fig. 12 can be used. For positive feedback Fig 12(a) is used, based on the analysis circuit in Fig. 2. This method allows the designer to find $K_d(f)$ by selecting $V_1 = 10$ mV. It is also possible to plot the $K_d(f)$ curve by means of a standard frequency-reponse plotter (on paper or cathode-ray tube). Fig. 13 shows the $K_d(f)$ curve made by such a plotter, connected as shown in Fig. 12(a).

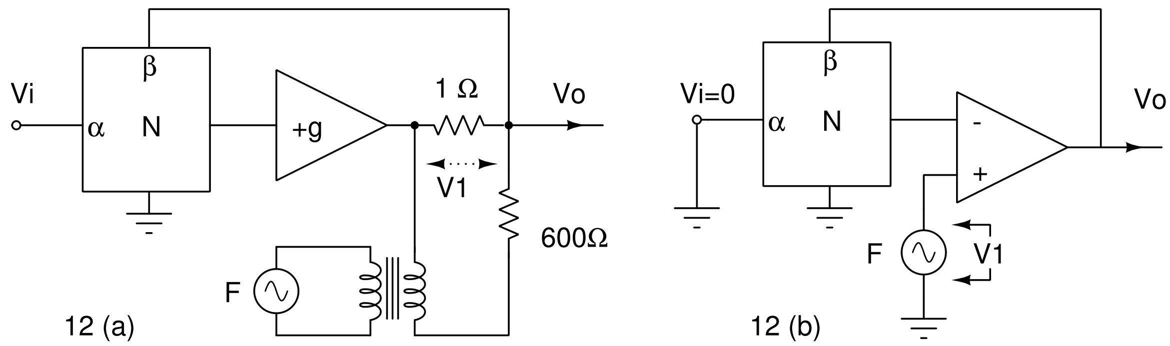


Figure 12: Analog experimental method used to plot $K_d(f)$ (a) Positive feedback, (b) Negative feedback

For negative-feedback filters the configuration shown in Fig. 12(b) is used. This circuit enables the designer to obtain the value of $1/\beta(\omega)$ directly. Here it is also possible to plot the output. Fig. 14 shows the superimposed curves of $K_d(f)$ for the same filter in the positive- and negative-feedback configuration.

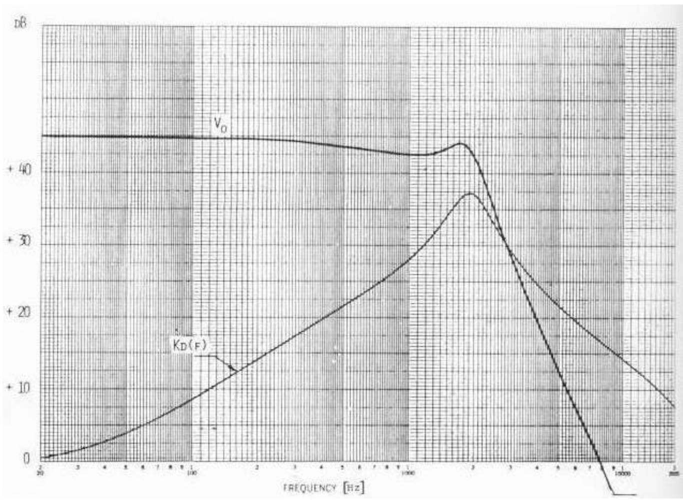


Figure 13: Plot of $K_d(f)$ for third-order Chebyschev filter $F_c = 2\text{kHz}$; $\epsilon = 2$ dB

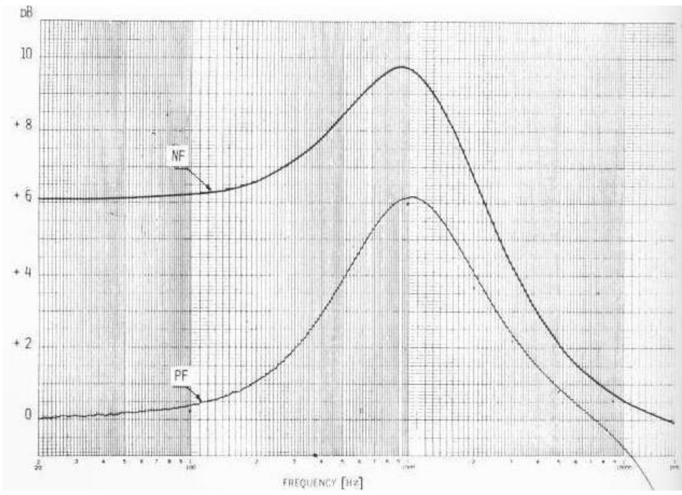


Figure 14: Plot of $K_d(f)$ for second-order positive- and negative-feedback Butterworth filters $F_c = 1$ kHz

6 CONCLUSION

The concept of the distortion multiplication factor has been analysed, and a simple definition is given of its wide application. We have derived equations that enable us to calculate K_d as a function of Q, either accurately or with sufficient approximation. We have demonstrated how to calculate Q_{max} so as to build a filter. The last two concepts enable us to predict the distortion that will be obtained from any type of filter with the most common circuit configurations. Finally, we have introduced three ways of finding the K_d value of an active filter. The first method is based on the resolution of a network, by means of a computer; the second is analytical, using Eqs. (3) and (4); and the third is experimental, based on a very simple measurement (Fig. 12).

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