

Elliott Antokoletz
Michael von Albrecht
(eds.)

International

Journal

of

Musicology

Vol.5

Offprint

1997



PETER LANG
Europäischer Verlag der Wissenschaften

Zoltán Göncz

Reconstruction of the Final Contrapunctus of The Art of Fugue

*And when they ask us what we're doing,
you can say, We're remembering.*

Ray Bradbury, Fahrenheit 451

Zusammenfassung: Die Bachsche Fuge ist die komplexe, aber gesetzmäßige und planvolle Entfaltung der kontrapunktischen Kombinationen der exponierten Themen und Kontrasubjekte. Im Contrapunctus 14 sind die kontrapunktischen Kombinationsmöglichkeiten so streng festgelegt, daß sich das musikalische Material fast von selbst in seiner Eigengesetzlichkeit entwickelt. Konsequenterweise zu Ende gedacht, läßt es Zahlenproportionen und räumliche wie zeitliche Symmetrien erkennen. Wie man aus den verschiedenen Stretta-Kombinationen ersehen kann, sind die vier Themen (1, 3, 4, 4^{mv}) verwandtschaftlich aufeinander bezogen: In geeigneter Stellung, kombiniert mit Thema 2, können sie einander ersetzen. Die organische Einheit der Bachschen Fuge erreicht im Contrapunctus 14 ihren Höhepunkt: Diese äußerst verdichtete Struktur zieht die Summe aus dem gesamten Zyklus. Darüber hinaus bringt der letzte Contrapunctus einige neue formale und kontrapunktische Mittel ins Spiel. Das wichtigste von ihnen ist die 'Permutations-Matrix'; sie ist in die Exposition der ersten drei Themen 'einprogrammiert', offenbart sich in der Engführung dieser Bauelemente und verleiht dem Zyklus enzyklopädischen Reichtum.

Introduction

The present essay appears as the last phase of research extending over several years, and tries to reconstruct in full the fragmentarily surviving final fugue of J. S. Bach's *The Art of Fugue* (Contrapunctus 14 = BWV 1080:19).^{*} It sums up the results achieved thus far and, by reconsidering them, draws conclusions. At the same time, it aims at keeping the reconstruction as close to the composer's original concept as possible. One of the prerequisites for the successful implementation of the work was the existence of a reliable philology in this matter, giving authentic answers to several questions concerning the cycle

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and settling successfully the uncertainties prevailing for more than two hundred years.

From the point of view of reconstruction, one of these important issues focused on the possible performance of the work. The four-stave-score notation of the first edition left room for uncertainty regarding the planned instrument(s) although the final fugue is notated on two staves in the appendix to the autograph manuscript. Thorough investigation of the spacing showed beyond a doubt what passed for self-evident in Bach's time, namely that, with the exception of the two mirror fugues, the movements were originally written for a keyboard instrument, two hands.¹ The original sequence of the cycle, consisting of 14² movements in the earlier variant and of 14+4³ in the later one, has essentially been solved, though it does not seem to be decisive concerning the continuation. Nevertheless, the numerology and the special construction of the final movement will receive special emphasis in the wider context of the whole series.

The most essential condition to achieve, that the work could come near to the original form planned by Bach, lay in the final item itself. From a certain point of the fugue onwards (the statement of the fourth subject),⁴ several theoretically equivalent and correct solutions could have been conceivable both with regard to counterpoint and form, considering the solutions of the other movements of the cycle. Owing to a strange stroke of luck, there appears a construction in this place of insecure formal branching off which has been unique in the history of music ever since, i.e. the permutational matrix which, apart from originating authentically with Bach, can be proved to have been ready at the time of the genesis of the work (that is, earlier than the surviving section).

In the course of the continuation it meant considerable help that the fugue bears the extremely strict archaizing features of *stile antico*,⁵ excluding thereby almost all non-thematic, non-organic elements and bringing the contrapuntal treatment into the focus which appears in this movement more concen-

1 Heinrich Husmann, "Die Kunst der Fuge als Klavierwerk", *Bach-Jahrbuch* (1938). Gustav M. Leonhardt, *The Art of Fugue, Bach's Last Harpsichord Work* (The Hague, 1952).

2 Douglas Seaton, "The Autograph: An Early Version of the 'Art of Fugue'", *Current Musicology* no. 19 (1975), pp. 54–59.

3 Gregory Butler, "Ordering Problem in J. S. Bach's Art of Fugue Resolved", *The Musical Quarterly* lxxix (1983), pp. 44–61.

4 Zoltán Göncz, "Bach kontrapunktikus labirintusa" [Bach's contrapuntal labyrinth], *Magyar Zene* (1990/2), pp. 134–139.

5 Christoph Wolff, *Der stile antico in der Musik Johann Sebastian Bachs* (Wiesbaden, 1968).

trated and complex than the oeuvre in general. The describable, written down part is necessarily much shorter than the entire reconstruction process. It does not contain all the reconstruction stages which proved to be dead-ends, erroneous paths as I progressed in the work. They would have unnecessarily burdened the volume of the essay and made it impossible to survey. They are referred to in the most essential, most instructive cases only.⁶ The present paper does not treat at length the attempts made so far at completing the movement, nor their criticism. Others have already performed it exhaustively.⁷ It mentions only some typical deficiencies and makes positive references to the insufficient nature of these attempts.

In view of the fact that in literature the formal (contrapuntal) solutions of the surviving portion of the movement are interpreted from many-sided and often very different angles and that certain phenomena are either judged erroneously or not taken into consideration at all, it seemed practical to begin with the analysis of the surviving 239 measures. All the more so as germs of innumerable solutions that unfold in their full complexity in the section after the tearing already crop up in the earlier sections.

1. The Analysis

1.1. The First Section

Axial symmetry is a rare morphological feature of the lapidary first subject of the fugue moving, for the greater part, in minim values. From the central fourth note onwards the melody consisting of seven notes repeats the first three notes in retrograde motion. (Of course, this peculiarity occurs only in the dux form.) The comes variants appearing in the second and fourth entries of the exposition (mm. 6 and 11) constitute a tonal answer: the entry of the dux at the upper fifth is modified to a fourth here. In the following it can be observed that the first two notes and the last one of the first subject are most ready to change under the influence of the harmonic and melodic environment.

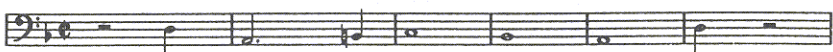
6 Resulting from the work in progress on the nature of the reconstruction process, the papers published later override at certain points the earlier formulated hypotheses. Such is the relationship between the paper mentioned under 4 and the one edited later (Göncz, "Bach – a szerialista?", *Magyar Zene* (1990/3) pp. 317–322). The edition which arises from combining the previous two essays (Göncz, "The Permutational Matrix in J. S. Bach's Art of Fugue", *Studia Musicologica* 33 (1991), pp. 109–119) naturally does not contain the assumptions overruled later.

7 Walter Kolneder, *Die Kunst der Fuge. Mythen des 20. Jahrhunderts* (Wilhelmshaven, 1977), pp. 301–327. Erich Bergel, *Bach's letzte Fuge* (Max Brockhaus Musikverlag, 1985), pp. 196–203.



Example 1

After the exposition the inversion of the subject appears in the bass (m. 21),



Example 2

which overlaps with the original form (m. 24) in the tenor voice. After the statement of another subject in inversion (m. 30: alto voice) the comes and dux forms of the subject in the original overlap at an interval of one measure from each other (m. 37: soprano-alto). This stretto (1+1) will be of decisive importance later on as well: it occurs not only in the first section (m. 61) but also in the stretto of the second section (m. 182), combined naturally with the second subject here and later on as well, as is going to be demonstrated.



Example 3

In the stretto (m. 71) following the appearance of the original (m. 43), the inverted formulae (m. 55) and a further 1+1 combination (m. 61), two forms in inversion are copied onto each other ($1^{inv}+1^{inv}$).

This combination is practically identical with the inversion of the above one (1+1).



Example 4

In the stretto beginning at m. 79 ($1^{inv}+1$) the inverted and original positions are present; the imitation takes place at the interval of two measures here.



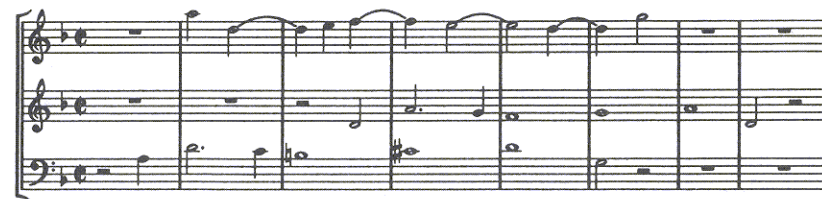
Example 5

In the subsequent stretto (m. 89) there are already three overlapping subjects. It is interesting to note that the imitating second voice (tenor) enters as a subject in inversion but continues in the original form ($1+1^{var}+1$) from m. 92 onwards when the third subject enters (alto).



Example 6

In the last combination where three subjects are also entering in close succession (m. 97) the inverted imitation of the second voice (modified to some extent in m. 101) follows the subject in the original position by half a measure (*imitatio per arsin et thesin*) which is further condensed by the third voice in which the subject in the original position can be heard ($1+1^{inv}+1$).



Example 7

The first section closes with the subject in the original position, appearing alone in the bass (m. 105).

The subject entries touch two mediant keys (F major, B flat major) in addition to the three principal keys (D minor, A minor, G minor). In the dramaturgy of stretti the principle of contrapuntal climax comes into force: the simpler combinations are followed by increasingly complex, thematically more

and more condensed ones. With regard to subject formations it may be observed that the two forms (original and inverted) occur at least once in each voice.

		1			1			1	1 ⁱ	1 ⁱ		
		1			1 ⁱ	1			1 ⁱ	1	1	1
	1			1			1 ⁱ	1 ⁱ	1 ^v	1		
1			1 ⁱ			1	1		1	1		

Example 8

1 = original

1ⁱ = inverted

1^v = original/inverted

The next example lists thematically the stretti of the original and inverted forms of the first subject. In order to facilitate comparison of the stretto positions, all subjects are given in identical position, in a transposition adjusted to the initial subject.

Example 9 consists of two systems of musical notation. Each system has four staves. The top staff of each system contains a melodic line with various rhythmic values. The other three staves in each system contain accompaniment. On the right side of each system, there are four arrows pointing left towards the staves, indicating specific points of thematic material or stretto entries.

Example 9

1.2. The Second Section

Characteristic of the rhythmic profile of the second subject is a continuous motion in quavers. The framework of the seemingly capricious winding melodic web is based on a sequence of thirds stepping upwards by seconds. This direction coincides with the ascending second part of the first subject.)

Example 10 consists of two systems of musical notation. Each system has three staves. The top staff of each system contains a continuous melodic line with a sequence of thirds stepping upwards by seconds. The other two staves in each system contain accompaniment.

Example 10

At the beginning of the exposition of the second subject (starting at m. 114) it can be observed how carefully Bach spans the formal boundary between the two sections by a 4–3 suspension and extending the cadential chord, respectively, almost hiding the announcement of the new subject.⁸

After the episode following the four entries of the exposition, the earlier stated and elaborated first and second subjects are combined (m. 147) (1+2^a).⁹

Formally the first two stretti can also be understood as the counter-exposition of the second subject because the (redundant) soprano in which the comes is heard in the exposition now renders the dux here while in the alto it happens vice versa.

⁸ The autograph shows that the end of the first section would originally not have overlapped the opening exposition at such length.

⁹ The superior letter [^a] in the symbol of the stretto serves for distinguishing the stretti, which have identical thematic material but receive different treatment.



Example 11

d = (second subject) dux
c = (second subject) comes

	c		
d			
			c
		d	

d	
	c
	1
1	

Example 12

This phenomenon of elision deserves particular attention because a formal ambiguity and compression of this kind will remain in force further on as well, and also will assume extreme proportions.

In the third (F major) stretto (m. 167) (1+2^b) the first subject enters one measure later than in the first two strettos (1+2^a).



Example 13

In the last (fourth) stretto of the section (m. 180) the first subject enters in a position corresponding to the previous entry (1+2^b), followed by another first

subject one measure later (1+1+2). The relationship of the first two overlapping subjects agrees with the stretto (1+1) which was heard in the first section several times (mm. 37, 61). Just as in the last strettos of the first section the thematic condensation reaches a density of three parts.



Example 14

The four entries of this section that join the first and second subjects demonstrate three different strettos of the two subjects (1+2^a, 1+2^b, 1+1+2). (The following table shows – similar to ex. 9 – the subjects in identical key and position.)



Example 15 a

Example 15 b

The four entries of the section introduce the three principal minor keys and the mediant F major. In this section the two subjects appear at least once in each voice.

2		1	1
	2		1
	1	2	
1			2

Key: d a F g

Example 16

It can be seen that the modulating episodes, but also the one-measure codettas of the exposition, are each constructed from the downward winding motif of the last measure of the second subject. In the modulating episodes of the 88-measure second section this characteristic direct material occurs everywhere, except for mm. 174–175: in m. 189 in inversion and immediately before the G minor cadence concluding the section (m. 191) also in simultaneous inversion.

1.3. The Third Section

The contours of the chromatic third subject, which move, for the greater part, in minims, greatly resemble the bare framework of the second subject. This is by no means surprising if we consider that these relationships form the basis on which these rhythmically and melodically diverging, moreover contrasting, subjects can establish varied contrapuntal combinations with each other.

The third subject overlaps with itself already in the exposition (stretto exposition) (mm. 193–205) (3+3^a).

Example 17

A novelty here is the appearance of the inversion of the subject (m. 213),

Example 18

which is followed by a stretto of dux and comes forms in the original position (3+3^b) (m. 217) at a closer time interval (of half a measure) than that of the exposition (3+3^a).

Example 19

After the announcement of the subject in inversion in the bass (m. 222) another imitation of the subject follows in close succession (m. 225), in which the dux imitates the comes by the displacement of three crotchets (3+3^c). Similar to the last stretto of the first subject (m. 97), the accented and unaccented notes of the subject are inverted in this stretto (imitatio per arsin et thesin).

Example 20

By fully exploiting the contrapuntal possibilities, three different kinds of stretto are realized in the exposition and development of the third subject (3+3^a, 3+3^b, 3+3^c).

Example 21

Following the imperfect cadence concluding the exposition and development of the third subject, respectively, there begins the weaving together of the first three subjects which is interrupted after the first entry (1+2+3).¹⁰

Example 22 a

10 This stretto occurs in the autograph manuscript only. The printed edition stops namely at m. 233 to mitigate the torso character.

Example 22 b

3 = original
3ⁱ = inverted

		3						
	3					3		2
3					3			3
			3					1

Example 23

2. General Issues of Continuation

2.1. The Size

In connection with the continuation, the question arises as to how long Bach may have planned the movement to be originally. In literature attempts have been made from many, fairly different angles to find an answer to this problem.

Many have supposed that symmetries of size prevailed within the movement and the cycle, respectively. In Graeser's view the size of the concluding quadruple fugue would have been identical with the sum of the number of measures of the four opening simple fugues (78+84+72+138=372).¹¹

Bergel supposes that the axis of symmetry of the closing *Contrapunctus* can be found between the second and third sections. Thus the second section would agree with the size of the third section, the first section with that of the

11 Wolfgang Graeser, "Bachs Kunst der Fuge", *Bach Jahrbuch* (1924), pp. 1-104.

fourth one. Consequently, the number of measures of the whole movement would lie in his opinion around $(114+80+80+114=)$ 388.¹²

By investigating the surviving copies of the first edition, G. Butler came to an interesting conclusion. He discovered that some of the completed copper engraving plates were renumbered. The missing page numbers which emerged on restoring the original sequence constitute in his assumption the number of measures of the closing section of the fugue, which can thus be supposed to be 279.¹³ Certain constructional considerations occasion doubt about this result, however.¹⁴

On reconstructing a contrapuntal work it is particularly hazardous to follow an *a priori* established number of measures because a too restricted size would not allow certain contrapuntal combinations even if it would be possible from the point of view of counterpoint and vice versa. The excessive length would necessarily result in increasing the redundant elements which contradict organic and economic development. In principle, the striving for symmetry (or symmetries) of size cannot be, of course, excluded. Nevertheless, it is fairly improbable that the development of contrapuntal construction would be subordinated to it.

In sum: the size of the final *Contrapunctus* can be postulated very cautiously before the actual contrapuntal elaboration takes place [see note 14].

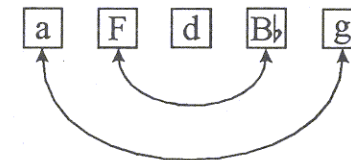
¹² Bergel 7, p. 169

¹³ Butler 3

¹⁴ In the various sections each subject (subject inversion) must appear at least once in each voice. This refers not only to the final *Contrapunctus* but to the other four fugues of the cycle as well, which consist of several subjects. In the case of the final fugue this means that the combination passages of the sections must contain a minimum of four entries to ensure that the subjects are announced in all four voices. It follows from it that in the third section (after the unfinished m. 239) there ought to have been at least three more entries and subsequently, in the fourth section which – as we are going to see would have included the inversion of the fourth subject as well – at least eight, altogether eleven entries. If we assume, taking the longest, second subject as a point of departure that the various entries had six measures each, the completion of the eleven entries would need $(11 \times 6 =)$ 66 measures. But then neither a modulating episode nor a codetta could take place between them. The thus "gained concentration" which proves naturally to be useless in such a schematical way during the actual formation, would extend the final *Contrapunctus* to 305 measures (according to Butler to 279 measures) and in reality even longer. It is not by chance that Butler's hypothesis of the size worded in his otherwise brilliant paper could not take root in literature.

2.2. The Form Design of Modulation

It is characteristic of all fugues of the cycle, thus also of the final *Contrapunctus*, that the entries of subjects are announced in the main key and the keys of the upper and lower fifth, respectively,¹⁵ and of the mediant keys, and frequently touch F major and B flat major as well. It is interesting to note that except for one such entry in *Contrapunctus* 4 (mm. 31–35) C major is the only one of these parallel major keys (i.e. the parallel key of A minor) not being present which is probably due to the prevalence of the principle of key symmetry.¹⁶



Example 24

The final fugue is governed by a particularly strict modulatory logic. It has emerged from the preceding analysis that the three main keys occur in the stretto portions of the various sections by all means. Besides, one or more entries touch the parallel major keys as well.

The principle of shunning repetitions comes into display in the keys of the subsequent subject entries: the entries in the immediate vicinity are in the rarest of cases in identical key. As a result, the entries are almost always linked by means of modulating episodes and codettas.

2.3. Interchanges of Voices

When assessing the size, the rule has already been established that the subjects are uttered at least once in each voice within the various formal sections.¹⁷ This practice so natural in expositions becomes perfectly active in stretto sections as well, and comes consistently into play not only in *Contrapunctus* 14, but in all multi-subject fugues of the cycle (*Contrapuncti* 8, 9, 10 and 11). In the four entries that interweave the first two subjects of the final fugue (mm. 147–193), just as in the section developing the first subject and its inversion

¹⁵ Except for the E minor entry of *Contrapunctus* 11 (mm. 146–150).

¹⁶ Cp. Bergel 7, pp. 206–207.

¹⁷ Cp. Bergel 7, pp. 171, 178.

(mm. 21–114, see above), it can also be observed that the interchanges of voices are carried out by exploiting the most varied contrapuntal possibilities.

2.4. The Spacing

It makes the authenticity of several attempts at completion dubious that the keyboard spacing is left out of consideration.¹⁸ The typical keyboard spacing comes about as the result of the interaction between two requirements. (The range of the instruments of the time determines the lowest and highest notes from the beginning.) The distance of the voices from each other is determined by the technical limitations of the playing in both hands. In the case of a four-part construction when each hand is entrusted with two parts, the maximum range between the bass-tenor and alto-soprano cannot extend over a tenth.¹⁹ This limitation is, of course, not valid when one hand has three parts, because then the other hand is capable of moving to a greater distance, at least in principle.²⁰ It goes without saying that upon unfolding two voices together, the tenor-alto interval may exceed the tenth, since both hands can move away from each other at discretion.²¹

While the technical restrictions of playing the manuals only keep together the voices, the requirement of polyphonic transparency does not allow the interval between them to become too narrow, otherwise the contours of the individual voices do not emerge plastically. In the same way, such spacing between the voices should also be avoided where they cross for a longer period of time, primarily because their identity would become doubtful and the polyphonic texture blurred.

A typical keyboard spacing comes into existence as the result of the interaction of centrifugal endeavours²² (to keep a certain distance) for transparency and of gravitational efforts to be able to play it by two hands, and this double requirement secures its balance.

18 This erroneous concept was kept alive by the keyboard score manner of notation (*Klavierpartitur*) and the opinion that since the cycle was abstract to such extent, the composer could not possibly have thought of its instrumental performance. Others held the view that the grandiosity of the work (particularly of *Contrapunctus 14*) comes into full display on the organ exclusively although the melodic line of the bass cannot be claimed to be pedal-like at all.

19 Bach uses this interval exclusively when the contrapuntal texture makes it unavoidable. Cp. Leonhardt 1, p. 19.

20 In *Contrapunctus 14* the left hand is entrusted with three voices in mm. 74, 79, 99, 104 and 211, the right hand in mm. 96, 180 and 185.

21 Such places in the final fugue are e.g. mm. 39, 110–112, 183 and 186–187.

22 One of the typical devices of intensification is extending the spacing.

3. The Reconstruction of the Third Section

3.1. The Stretti of the Three Subjects

The spacing, the form design of modulations, the study and awareness, then extrapolation of the characteristic traits of interchanges of voices do not provide sufficient evidence for the reconstruction. A fugue – and particularly a quadruple fugue – formally is nothing else but the contrapuntal elaboration and development of subjects and counter-subjects of increasing tension. The designation *Contrapunctus*, which is unique in the lifework, stresses placing contrapuntal texture into the foreground. Consequently, to continue the stretto of the third section one must necessarily be aware of the contrapuntal combination possibilities, in which the three subjects can be combined, apart from the one extant in the entry beginning with m. 233 (1+2+3).

A thorough investigation of the contrapuntal potentials of the stretti formed by the three subjects reveals that there are two different solutions for increasing the thematic concentration. One is when the third subject is doubled in a way that it overlaps with the above (1+2+3) combination as a fourth voice (1+2+3+3), in fact, in the same rhythmic position (*imitatio per arsin et thesin*) as in the third stretto of the third section (3+3^c mm. 225–230), but in a different pitch relation.



Example 25

The plasticity of this combination comes out particularly in the major key because there the third subject, displaced by three crotchets, can retain its characteristic chromatic head motif (Ger. B-A-C-H = B flat-A-C-B), in contrast with the minor entry where this is impossible without the violation of the style. It is obvious that in this stretto all four voices already state a subject, in which the thematic density reaches the original number of voices, and no free voice is available.

It follows logically from the above that in the other stretto of the three subjects the first subject is doubled (1+1+2+3) in a manner that both the rhythmic and the pitch relationships of the two first subjects are identical with those of the first (1+1) and second sections (1+1+2).²³ The thematic concentration affects four voices in this instance as well. Moreover, it calls forth a fifth one. Due to overlapping, the semibreves and minims of the subjects in stretto bring about the interruption of the continual quaver motion, of rhythmic continuity. The fifth, supernumerary voice is built into these rhythmic "holes", overcoming the hiatus.



Example 26 a

Compared with the stretti of three subjects, it appears that the voices (1+1+2+3, 1+2+3+3) attached to the "basic stretto" (1+2+3) are in melodic relationship with each other.



Example 26 b

The above relationships make evident – and that will be even more significant later on – that the subjects (and counter-subjects, respectively) are each other's variants in melodic and contrapuntal sense alike and that certain of their segments are capable of replacing each other when they get into an appropriate position.

The continuation of the section, which causes the overlapping of the three subjects from the interrupted m. 239 onwards, can be compared with working out an equation with several unknowns in which the change of any one of the parameters changes the final result and the rest of the parameters. Though most of important parameters – the three possible stretti, the keys which can be and should be run through as well as the smallest number of entries resulting from the "obligatory" interchanges of voices – are known, still the *succession* of the subject entries is uncertain. The same applies to the sequence of keys and interchanges of voices of the subject entries and even to the number of entries, which can extend above four if the contrapuntal possibilities allow it.

The possibilities are, however, all but unlimited; so much better for the reconstruction. The possibility of interchanges of voices is considerably restricted by the fact that the numerous pitch exchanges resulting from the three kinds of stretti are excluded. Any arrangement of the 1+1+2+3 stretto, in which the second or third subjects are announced in the bass leads to an unacceptable harmony (e.g. a stressed six-four chord).

²³ It is characteristic of other fugues of Bach's contrapuntal art as well that later stretti include earlier ones or make them overlap.

Example 27 is a musical score consisting of two systems. The first system contains five staves, and the second system contains four staves. The notation includes various rhythmic values, accidentals, and slurs, illustrating a complex polyphonic texture.

Example 27

It follows implicitly that the second and third subjects can occur exclusively in the bass in the other two combinations only (1+2+3, 1+2+3+3). Other conjunctions are excluded by the restricted quality of polyphonic transparency. E.g. in the following case there would come forth undesirable long crossings of voices and too narrow passages.

Example 28 is a musical score consisting of three systems. The first system has four staves, the second has three, and the third has three. The notation includes various rhythmic values, accidentals, and slurs, illustrating a complex polyphonic texture.

Example 28



Example 29

Even the possible sequence of the stretti is determined by the fact that due precisely to the increased number of parts the five-part combination (1+1+2+3) must have come at the end of this section. From it derives that the sequence of the three kinds of stretti looks like this in the function of the increasing thematic concentration and the number of parts, respectively: 1+2+3, 1+2+3+3, 1+1+2+3. On the other hand, it must be remarked that the manner of design and the direction of the modulations of the episodes constructed (similar to the second section) from the final motif of the second subject are fairly determined. The episodes are not mere passive constituents (filling material) of the fugue structure, but establish the form actively, on equal terms with the subject entries. The modulating sequences beginning after the individual subject entries run into the following entry almost on their own and start the subjects automatically, defining at the same time the key sequence of the thematic statement by the direction of their modulations.

Owing to the strict logic of the fugue construction, the continuation conceived by the composer can be approached the more we succeed in realizing that the contrapuntal potentials inherent in the subjects are at the point of intersection of various construction principles, rules and tendencies. These principles, criteria and rules of composition severely limit the feasibilities of continuation and indicate paradoxically the apparent narrow path of implementation on which the composer ought to have gone. The reconstructed section appears in the intersection of the above enumerated formal expectations, contrapuntal feasibilities, restrictions and prohibitions of these factors and laws as their *optimum coincidence* and *synthesis*.

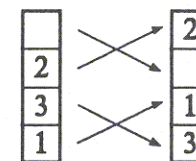
3.2. The Combination Passage of the Third Section

In m. 239 the codetta which continues the unwinding motif of the second subject modulates into A minor. In the stretto beginning with m. 240 the second subject gets into the soprano; its head motif is imitated by the alto an octave lower. This imitation corresponds to the one in m. 233, but there the soprano imitates the second subject uttered in the alto (invertible counterpoint at the octave).²⁴



Example 30

The first and third subjects appearing in the two lower parts exchange places similarly.



Example 31

The modulating sequence commencing after the combination leads into a B-flat major entry which realizes a more complex manner of the stretto of the three subjects (1+2+3+3) discussed earlier. All four voices are thematic: the first subject gets into the soprano, the second into the bass while the third subject overlaps by an exchange of accents in the two inner voices (imitatio

²⁴ In m. 233 the second subject and its imitation are extended at the beginning by a quarter to span the rhythmic gap between the imperfect cadence and the entry of the subject.

per arsin et thesin). One of the favourable consequences of such an arrangement of subjects for playing technique is that while the right hand has the task to produce the subjects in slower time values (the first and the two third subjects), the left hand, remaining free, intones the lively second subject easily.

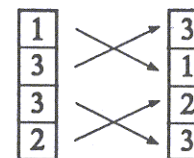
The major key makes possible the real intonation of the chromatic head motif of the third subject with a shift of accent, which includes as a result of the B-flat tonality, the notes B-flat-A-C-B [Ger. B-A-C-H]. Of course, it is not this fact but the principle of avoiding repetitions that justifies the B-flat major tonality inasmuch as the major entry of the second section (mm. 167–174) was in F.

The slight alteration of the end of the third subject heard in the tenor (m. 254) is brought about by the necessity to avoid consecutive fifths between the bass and the tenor occurring exclusively in the major key when the third subject lies above the second one.

Example 32

The new entry after the modulating episode (m. 261) is in G minor, the third principal key, and utters also the stretto doubling the third subject (1+2+3+3). Compared with the previous entry – and similar to the first two entries – the subjects exchange octaves in pairs here as well.

The beginning of the second subject rendered in the tenor is embedded into an imitational environment, as in m. 233.



Example 33

Example 34

Due to the new key and the interchanges of voices, the stretto is not a dull repetition of the preceding one: the voices are heard in an extremely extended position to be played with two hands yet, and in the two outer voices the two third subjects, imitating each other, come plastically into relief. With this fourth entry the formal requirement is fulfilled, according to which the subjects are supposed to appear in each voice. Moreover, even the three principal keys are heard, one of them enlarged by one of the mediant (B-flat major).

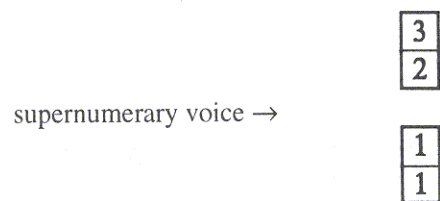
	2	1	3
2		3	1
3	1	3	2
1	3	2	3

Keys: d a B_b g

Example 35

Evidently the formal section does not end here. The stretto is still to come in which the first subject is doubled (1+1+2+3). This fifth entry (m. 269) is again in the main key. The two first subjects overlap in the two lower voices,

the second subject occurring in the alto, the third one in the soprano, while the fifth, supernumerary voice is wedged between the tenor and the alto.



Example 36

The use of supernumerary, non-thematic voices – mainly at the end of fugues – is all but a rare solution in Bach's art of counterpoint. Several examples can be found in the other *Contrapuncti* as well.²⁵ The difference is that this increase of the number of voices is justified by the heightening of grandiosity through pedal point in the other cases, whereas the appearance of a supernumerary voice here has explicitly contrapuntal reasons (see above).

The hidden tendency of the growth of the number of voices can already be observed in an earlier passage of *Contrapunctus* 14, at the end of the second section, where a thematically saturated combination can be heard (1+1+2) and the free voice falling well-nigh apart in a successive manner condenses the structure to a latent five-voice one.



Example 37

²⁵ Supernumerary voices can be found in the last measures of *Contrapuncti* 5, 6, 7 and 11. This solution occurs frequently in the organ fugues, thus at the end of several fugues of the *Well-Tempered Clavier*.

The question may naturally arise whether the above section could not be condensed more than above, to a minimum of four entries. In principle, it could be done by omitting one of the preceding entries so that its key would be taken over by the five-voice entry. This procedure is only a theoretical consideration. When reconstructing this section, it was namely an important aspect that the formal expectations (interchanges of voices, key scheme, etc.) should not be carried out "on paper" only but realized – as mentioned before – through the most favourable, most plactical rendering of the various contrapuntal combinations. In the case of the five-voice stretto this optimum (the feasibility to play with two hands, shunning voice-crossing, transparency, plasticity) can best be achieved through the above spacing. This manner of spacing could not substitute for any of the earlier – for that matter, similarly "optimized" – entries since there would remain subjects in each case which would not be heard in some of the voices.

What has been said demonstrates perfectly how complicated it is to compose a fugue of several subjects, to reconstruct it successfully and how complex the task is. What makes the previous objection theoretical is, among other things, that the succession of the individual entries, the pitch exchanges of the entries, are strongly determined by the fairly idiosyncratically whirling expositions.

4. The Reconstruction of the Fourth Section

4.1. The Stretti of the Four Subjects

The section weaving together the three subjects is followed by the appearance of the fourth subject (the main subject of the cycle). Resulting from its torso character, this statement was by no means evident in the course of the history of acceptance of the fugue. Many (Hauptmann,²⁶ Spitta,²⁷ Rust²⁸) doubted that it belonged to the cycle *Fuga a 3 Soggetti*, calling attention to the fact that the main subject did not occur in it. Others (e.g. Bitsch,²⁹ Martini³⁰) saw in the first subject the simplified version of the main subject of *The Art of Fugue*.

²⁶ Moritz Hauptmann, *Erläuterungen zu Joh. Seb. Bach's Kunst der Fuge* (Leipzig, 1841), 41925.

²⁷ Philipp Spitta, *Joh. Seb. Bach* (Leipzig, 1873–80).

²⁸ Wilhelm Rust, *Vorwort zur Ausgabe der Kunst der Fuge durch die Bach-Gesellschaft*, Jhg. xxv/1 (Leipzig: Breitkopf & Härtel, 1875).

²⁹ Marcel Bitsch, "J. S. Bach L'art de la Fugue", *L'education musicale* 1971, pp. 164–168, 204–207.

³⁰ Bernhard Martin, "J. S. Bachs letzte Fuge", *Die Musik* 1941, pp. 409–412.

Gustav Nottebohm, the eminent Beethoven scholar provided the first substantial evidence in 1881³¹ which proved that the work formed an organic part of the cycle. His research into counterpoint made obvious that Bach conceived the movement as a quadruple fugue whose missing fourth subject would have been the main subject of the cycle.



Example 38

The above example (1+2+3+4) shows that in the final fugue there would have been a dotted variant of the main subject of the cycle extended by passing notes – which modifies to a certain extent Nottebohm's original concept – and not the lapidary one available in the four simple fugues and *Contrapunctus* 9, respectively.³²

31 Gustav Nottebohm, "J. S. Bachs letzte Fuge", *Musik-Welt* 1880/1881, pp. 232–236, 244–246.

32 This possibility was mentioned by Nottebohm as well (Nottebohm 31, p. 233). Further divergences between the present combination and the one in Nottebohm's study can be found in the pitch arrangement of the subjects. In Nottebohm's study the second subject is in the soprano and the fourth one in the alto, which makes it impossible that the spacing be played with two hands. There will be further reasons to prove that the pitch exchange shown in ex. 38 was conceived by Bach. (See permutational matrix.)

It is significant though not more conclusive than the contrapuntal considerations that Bach, who had attached great importance to numerology in his earlier works, too, must have planned to elaborate the fourteen-note variant of the main subject of the cycle at the end of his last work.³³

At certain points of *The Art of Fugue* the traces of later sections can be discovered. It is quite natural to find an abundance of agreements, similarities and overlappings in a monothematic cycle of such huge dimensions which the composer was working on for several years, returning to some of the movements and sections repeatedly. The reason of their similarities lies, beside being composed in parallel, in that the counter-voices are unambiguously determined by the main subject. The various alternating counter-voices, steady counter-subjects and subjects, respectively, all form a plastical background to the main subject, a kind of "negative".

This feature is true inversely as well. Nottebohm's resolution (i.e. the main subject) is hidden immanently in one of the extant segments of the work, in the last measures of the incomplete manuscript (P 200, *Beilage* 3). The three subjects develop the contours of the main subject copy like a photo negative.³⁴



Example 39 a

The above melodic-contrapuntal phenomenon completes the relationship between the subjects outlined earlier (cf. Ex. 27): all subjects are melodically and contrapuntally related with one another and, appearing at a certain pitch or time, they are capable of substituting each other by following one another's melodic contours.

33 H. G. Hocke, "Vorwort zu Johann Sebastian Bach die Kunst der Fuge BWV 1080", *Facsimile-Reihe Bachscher Werke und Schriftstücke* 14 (Leipzig, 1979).

34 Cp. Hans Heinrich Eggebrecht, *Bachs Kunst der Fuge* (Piper – Schott³1988, p. 12).

Example 39b

Example 40

On studying the invertible counterpoint possibilities of the four subjects, it turns out that almost any of the subjects may occur in any of the voices: the conditions of the quadruple counterpoint are given. A forbidden harmony comes into being only if the main subject gets into the bass. In that instance there emerges a six-four chord on the principal accent of the fifth measure of the stretto, which is not permitted in this style.

Example 41

There is still a possibility for the main subject to appear in the bass beside the quadruple counterpoint of the four subjects if the first subject exchanges places with the other three subjects at the twelfth and not in the octave ($1+2+3+4|2$).³⁵

The apparent prohibition of another pitch exchange should also be cleared here. If the second subject is in the bass, there emerges a six-four chord on the principal accent of the third measure of the stretto. The same applies to the situation when the second subject is in the tenor and the third one in the bass, since the third subject enters in the second half of the third measure of the stretto only. Thus the tenor is the lowest part at the beginning of the measure.

³⁵ Nottebohm's study includes these possibilities of invertible counterpoint (Nottebohm 31, pp. 234–235).

The breaking of the rule is merely apparent – due to a remarkable solution – because the second subject becomes a latently two-voice one in measures 2 and 3 of the stretto,³⁶ whereby a virtual basso part (pedal point) comes into existence which puts an end to the undesirable chord inversion. As mentioned before, several pitch exchanges are naturally limited by the technical capabilities of the two hands.

Example 42

In his monograph published in 1977 Kolneder³⁷ listed precisely twenty attempts at reconstruction. The majority are based on and depart from Nottebohm's resolution. A lot of scholars understand the words of Mizler's obituary literally³⁸ and include the inversion of all four subjects later on.³⁹ These com-

36 Several examples of such successive divisions of voices can be found – mainly in solo literature but also in the present cycle, cp. Leonhardt 1, p. 16.

37 Kolneder 7.

38 H. J. Schulze, *Bach Dokumente* vol. 3 (Leipzig-Kassel, 1972), p. 86. "Seine letzte Krankheit hat ihn verhindert seinem Entwurf nach die vorletzte Fuge völlig zu Ende zu bringen, und die letzte, welche 4 Themata enthalten, und nachgehends in allen 4 Stimmen Note für Note umgekehrt werden sollte, auszuarbeiten." [His final illness prevented him, according to his draft, from bringing to a complete end the penulti-

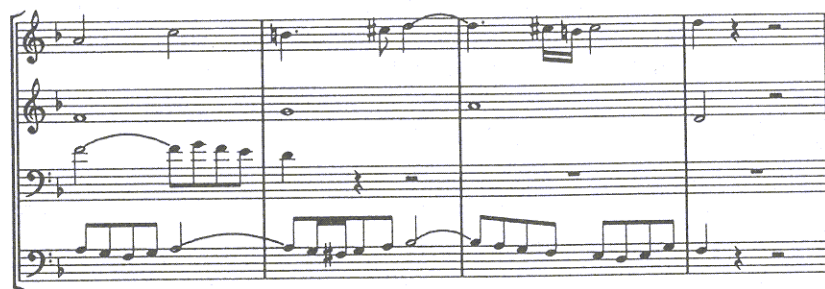
binations make a strange impression and lack the necessary force of conviction. Besides, the various completions frequently contain inadequate elements of style, forced stretti, inconsistencies of form.

Example 43 a

mate fugue and from working out the last one, which was to contain four subjects and to have been afterwards inverted note for note in all four voices.]

39 The combination created by the inversion of all four subjects is also contained in Nottebohm's study (Nottebohm 31, pp. 235–236).

Example a



Example 43 b

In connection with the subject formulae it can be safely claimed that following the interrupted m. 239 Bach did not intend to arrange the inverted form of the earlier-stated three subjects which confronted each other:⁴⁰ (1) The first subject is announced until the entry of the second subject (up to m. 114) sixteen times in original, eight times in inverted form and combined with the second and third subjects (in six further entries after m. 147) exclusively in the original position. (2) The rather bizarre inverted form of the second subject (nine entries) does not occur in the torso at all. (3) The proportion of occurrences of the two forms of the third subject is even more extreme than that of



Example b

40 Cp. Bergel 7, pp. 219–222.

the first subject: there are ten forms in the original position and two inverted forms. (4) In the fugues with several subjects in *The Art of Fugue*, the subjects stated and developed in the original and inverted forms figure – in the manner of a counter-fugue – in one form later on in the strettos created with the main subject. (*Contrapuncti* 10 and 11). In other words, even if the inverted form occurs before, it does not mean that it can be combined with the main subject.

As regards the fourth subject, it is worth considering, before we discuss the contrapuntal possibilities and survey the monothematic concept of the cycle, whether the following can be reconciled with the logic of the whole work. That is, while in the preceding fugues of *Contrapunctus* 14 both forms of the main subject (in the original position and inversion) appear on equal terms – of the initial four simple fugues two constitute the original, two are inversions, the three stretto counter-fugues contain both forms; of the fugues with several subjects items 8 and 10 contain exclusively the inverted form, item 9 only the form in the original position, item 11 both, and the two mirror fugues, of which the three-voice one is automatically a counter-fugue, contain naturally both forms – in the quadruple fugue concluding the set, the original form of the main subject occurs exclusively.

Departing from the form of the surviving fragments and considering the concept of the cycle with regard to the contrapuntal solutions of the finished movements, the arrangement of the main subject forms, and last but not least the stylistically alien character of the strettos which may be produced by inverting all four subjects, it may be supposed that in addition to the stretto described by Nottebohm (1+2+3+4) Bach also planned another stretto in the fourth section to be established by joining the four subjects, in which the inverted version of the main subject is associated with the forms in original position of the first three subjects (1+2+3+4^{inv}).

The contrapuntal combination of the four subjects is also possible under the above conditions.⁴¹

It may be observed that the inverted form of the main subject enters at an identical place compared to the second and third subjects of Nottebohm's combination in the original position, while the first subject is stated one measure earlier there, in the form of *dux* and in that of *comes* here. On collating the two combinations it emerges that two forms each of the main and first subjects almost substitute each other in certain places, following each other's outlines.

41 It seems advisable to clarify that in Bach's work neither the motion of voices leading from the augmented fifth to the perfect fifth (Ex. 38: m. 3, soprano-alto), nor from a diminished fifth to a perfect fifth (Ex. 44, mm. 3–4, bass-tenor) was considered as a forbidden parallel. The latter can be observed in m. 15 of *Contrapunctus* 6 between the soprano and alto as well as between the soprano and the tenor.

Example 44

Example 45

In the relation of the two strettii, the opinion worded in literature in another context according to which the first subject is one of the main subject variants⁴² seems to be verified in another perspective in that the similarities of the

42 Cp. Bitsch 29. Martin 30.

melody have contrapuntal consequences.⁴³ Several other signs point to the fact that in addition to the original form of the first three subjects Bach also intended to elaborate the inverted form of the main subject. In *Contrapunctus* 10 a similar but more tell-tale "route marking" track can be found than in Ex. 39 (where the free counter-voice outlining the contours of the main subject was to be seen).

Example 46

What makes the section particularly interesting is that it forms a kind of bridge between the earlier⁴⁴ and later drafts of the cycle. In the early variant of the manuscript to be dated to the first half of the 1740s (designated at that time "6th fugue" yet = BWV 1080:10a) the exposition of the main subject begins with the inverse form heard in the soprano – naturally in one voice. In the later draft available in print (*Contrapunctus* 10) it is preceded by the exposition and development, respectively, of the other subject. In order to retain the continuous quaver motion, complementary voices not extant in the earlier variant are added to the first entry of the otherwise unaltered main subject exposition. It may be assumed that the compositional date of these later-added voices and of the final *Contrapunctus* not available in the earlier draft yet can have been close to each other and may account for the motivic and harmonic similarities.⁴⁵

43 Cp. Exx. 27, 39 and 40, respectively.

44 Douglas Seaton, "The Autograph: An Early Version of the 'Art of Fugue'", *Current Musicology* no. 19 (1975), pp. 54–59.

45 Cp. Peter J. Williams, *J. S. Bach Kunst der Fuge. Preface* (Eulenburg, 1986), pp. xxi. xxxi.

By joining the two stretti (1+2+3+4, 1+2+3+4^{inv}) a new combination comes into being in which the simultaneously uttered original and inverted forms of the main subject as well as the other three subjects can be found (1+2+3+4+4^{inv}). Due to the above parallels,⁴⁶ the first subject enters in this case in a different place (invertible counterpoint at the twelfth) and the stretto is extended evidently by a fifth, moreover a sixth voice.⁴⁷

Example 47

46 Cp. Ex. 45.

47 In Nottebohm's study the two main subject forms sounding simultaneously and the stretto of the second subject (2+4+4^{inv}) are available, whereas the first and third subjects are missing in the combinations. Nottebohm 31, p. 245.

Example 48



Example 49

The appearance of the pedal point as sixth voice not only increases the grandiosity of the stretto and stresses the dissonances more plastically, but its employment has weighty harmonic reasons. On the principal accents of mm. 4, 5 and, in point of fact, 6 of the stretto there would be six-four chords if the pedal point did not neutralize them.

The simultaneous inversion of the main subject always took place in distinguished moments of the earlier *Contrapuncti* of *The Art of Fugue*, thus in the Picardy third coda of *Contrapunctus* 5 extended by supernumerary voices (mm. 86–90) as well as at the end of *Contrapunctus* 11 (triple fugue) (mm. 157–168). Based on the formal and dramaturgical solutions of these earlier appearances and because of the increased number of parts, it is most likely that the above simultaneous inverted stretto should come after the quadruple counterpoints which the four subjects form with each other (1+2+3+4, 1+2+3+4^{inv}).

After having taken stock of the stretti containing the first three subjects and the main subject in inversion (1+2+3+4^{inv}), then of the further overlapping with the main subject in the original position (1+2+3+4+4^{inv}) – just like during the course of reconstruction of the end of the third section – one must investigate the characteristic features of the pitch exchanges of these combinations.

In case of the four-voice stretto (1+2+3+4^{inv}) – similar to the last stretto of the third section (1+1+2+3) – any solution in which the second and third subjects are in the bass would lead to forbidden harmonies (accented six-four chord) and thus they fall completely out.

From this it follows logically that both the first and the inverted fourth subjects may occur in the lowest part. If we examine the possibilities further we may state that with neighbouring voices the following solution (inverted fourth subject – bass, second subject – tenor) cannot be said to be felicitous in which a voice-crossing arises in the course of a critical dissonance.

The arrangement below cannot be used either. In it the interval between the alto and the tenor is too big and it does not sound well when the appoggiatura-like dissonance occurs in the two lower voices.



Example 50

The relation of voices should also be excluded where the second subject and the inversion of the fourth subject are so close that their identity becomes insecure because of the crossings.



Example 51



Example 52

In them no forbidden harmonies emerge, no undesired crossings of voices occur. As a result, the spacing is rather airy yet void of large-range hiatuses between the neighbouring voices, and the texture remains to be played by two hands. In the lack of further considerations the sequence and key of these two entries as compared to each other are nevertheless uncertain. The possibility of pitch exchanges is still more limited, if that is feasible at all, with the stretto containing the five subjects (1+2+3+4+4^{inv}). The above pairing of voices (Ex. 51) by which the second and the inverted fourth subjects get too close to each other should be avoided here as well. The first subject cannot come above the main subject in inversion because it would result in consecutive fifths. This implies that the inversion of the main subject cannot figure in the bass, only in a higher voice. It has an unfavourable effect if the two forms of the main subject (4 and 4^{inv}) are rendered in an identical position because the steady interchanges of voices blur the contours of the melody, the subjects get mixed up and lose their identity.⁴⁸



Example 53



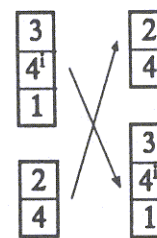
Example 54

To keep the contrapuntal texture condensed to six voices by means of pedal point in a range that can still be played by two hands raises even more difficulties than the combinations of the preceding one (1+2+3+4^{inv}). The above limitations (forbidden conjunctions, voice-crossings with bad effect, a range surpassing the span of the two hands) narrow down the pitch exchange possibilities of this combination to such an extent that only two meet the requirements of the numerous conditions.

⁴⁸ Cp. *Contrapunctus* 5, mm. 86–90, *Contrapunctus* 11, mm. 158–162.

Example 55 a

A comparison of these two pitch exchanges shows that the subjects become arranged into two blocks and these two blocks change pitches with each other (invertible counterpoint at the octave). The sequence and key of these two combinations are also uncertain – just like the possible pitch exchanges of the preceding two strettos (1+2+3+4, 1+2+3+4^{inv}) – for lack of further considerations.



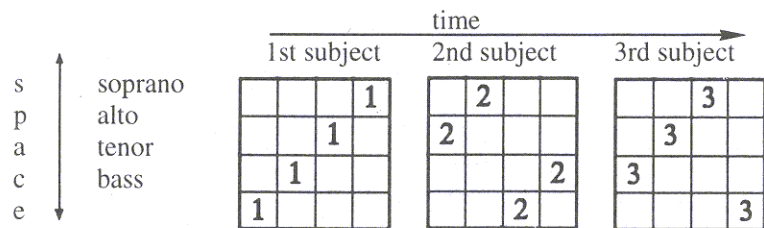
Example 55 b

4.2. The Permutational Matrix

As far as the appearance of the main subject is concerned, not only the pitch exchanges of the strettis with the other three subjects, their sequence, and keys are uncertain. Other problems concerning exclusively the form also emerge. It has not been clarified yet how the two forms of the main subject are elaborated, whether they receive an exposition on their own and if so, in what kind of a dramaturgy of form.⁴⁹ The facts described below answer the question with fairly great certainty how Bach planned the fourth subject, the main subject of the cycle to appear after the interrupted m. 239 (the further strettis of the three subjects).

The characteristic formation of the extant section of the movement reveals that the composer did not intend to state the main subject separately in the final *Contrapunctus*; it appears immediately together with the preceding three subjects. (The gigantic size of the movement and the monothematic nature of the cycle justify this dramaturgy.)

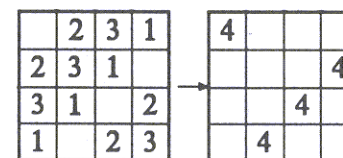
In the course of the statement of the first three subjects of *Contrapunctus* 14 (first subject: mm. 1–21, second subject: mm. 114–141, third subject: mm. 193–207), Bach applied a so-to-say serial sequence of voice entries decided in advance, by which he determined, in the terminology of the second half of the twentieth century, the space and time parameters of the subject entries.



Example 56

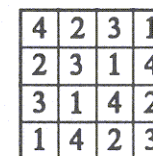
The superimposition of the three exposition matrices foreshadows and develops as a negative the sequence of the voice entries of the fourth subject.

49 Basically the possibilities could branch off in four directions: (1) The original and inverted forms of the main subject are stated separately (similar to *Contrapunctus* 11). (2) The two forms of the main subject appear alternately just as at the beginning of *Contrapunctus* 5. (3) The two forms of the main subject are stated simultaneously, in pairs, like at the end of *Contrapunctus* 5 and in the two entries of *Contrapunctus* 11. (4) The forms of the main subject are not stated separately (cp. *Contrapunctus* 9), the fourth subject is uttered immediately, together with the three other subjects.



Example 57

The copying of the four subjects onto each other displays a characteristic construction of Bach's oeuvre occurring mainly in the vocal fugues: that of the permutational fugue.⁵⁰



Example 58

The permutational fugue combines the fugue and canon structures, and its voices present the subjects extremely concentrated, without episodes and in a canon-like round. In relation to the quadruple fugue and the permutational passage, W. Neumann's statement deserves special attention: "Jede Permutations-fuge ist Tripel-, Quadrupel- oder Mehrfach-fuge, oder wenigstens ein Ansatz dazu."⁵¹ In *Contrapunctus* 14 the subject series of the permutational matrix (4-2-3-1) defines horizontally the events taking place in time and vertically the spacial layers: thus the (successive) temporal sequence of the subjects agrees with the (simultaneous) arrangement in space.⁵²

Bach's "serialism" determines not only the sequence of the subjects, but defines (with one exception) the precise pitch (position) of the subjects as

50 Werner J. Neumann, *J. S. Bachs Chorfüge* (Leipzig: Breitkopf & Härtel, 1950).

Zoltán Gárdonyi, *J. S. Bach kánon- és fúgaszerkesztő művészete* [J. S. Bach's art of canon and fugue construction], (Zeneműkiadó, 1972), pp. 122–132. Carl Dahlhaus, "Zur Geschichte der Permutationsfuge", *Bach-Jahrbuch* xlvii, 1959. Paul Walker, "Die Entstehung der Permutationsfuge", *Bach-Jahrbuch* 1989, pp. 21–41. (The latter study contains important information on the emergence of permutational structure in the instrumental literature before Bach as well.)

51 Neumann 50, p. 42

52 A similar solution can be found e.g. in cantata No. 50 (*Nun ist das Heil*) as well as in the final chorus (*Lob und Ehre und Preis und Gewalt*) of cantata No. 21 (*Ich hatte viel Bekümmernis*). In these two movements – just as in *Contrapunctus* 14 – the subjects move stepwise, ascending from below.

well. In the first entry of the permutational section the first three subjects appear at exactly the same pitch-level as in the first entry of their expositions. This solution continues to prevail consistently in the relationship of the subject expositions and the permutational matrix. (An exception is the first subject in the second entry which appears a fourth lower. This invertible counterpoint is justified by the striving to avoid a six-four chord on an accented place in m. 5 of the entry.)⁵³ The notes in the table below show the initial tones of the subjects.

	1st entry	2nd entry	3rd entry	4th entry
1st subject				



Example 60

Though this solution is correct in theory, it does not prepare the appearance of the fourth subject (the main subject of the cycle!) adequately. The opening becomes rather insignificant because the first note of the fourth subject coincides with the last note of the third subject heard in the preceding entry. It must be closer to the originally planned solution when the fourth section starts after the imperfect cadence of the third section. This is corroborated by two earlier portions of the fugue. The section of the final *Contrapunctus* with the entry of three subjects in close succession (mm. 232–233) begins after an imperfect cadence, but regarding the boundary of form, the similarity of a section of *Contrapunctus* 10 is even greater where the exposition of the inverted form of the main subject comes after the exposition of the first subject.⁵⁸

If the above solution of connection is applied, the close polyphonic texture breaks up before the entry of the fourth subject, and the appearance of the main subject, of the cycle receives as a result of that – and last but not least because of the harmonic emphasis – a dramaturgically equivalent accent.

4.3. The Inversion Matrix

It turns out from the comparison of the four strettos with "optimized" pitch exchange discussed above, which contain the inversion of the main subject (exx. 52 and 55), and from the study of the position of the inversion of the main subject that the inverted form of the main subject appears in four different voices.

⁵⁸ This section has earlier been mentioned from another aspect. Cp. ex. 46.

Example 61

2	2	3	4 ¹
3	4	4 ¹	3
	3	1	
1	4 ¹	2	2
4 ¹	1	4	1

Example 62

This feature coincides with the requirement according to which the individual subjects (subject inversions) must be uttered in each voice. This strange coincidence is only at the first glance surprising. The question is namely not that the perfection of the musical material is hindered by the laws of contrapuntal construction, but rather that the fullest and at the same time most economic evolvement of the subjects takes place by the operation of these laws, in fact, generating these regularities if viewed in the dimensions of the complete Bachian oeuvre.

The possibilities of establishing the form design of modulation of the section are fairly limited. It may be taken for granted that none of the above four strettis can be in the major key. The reason for it lies in the fact that on the principal accent of m. 3 of these subject entries the sixth and seventh degrees of this key appear simultaneously, and while this consonance in the minor key is an everyday phenomenon in this style, it is inconceivable in the major key.



Example 63

When one continues to consider the modulation process it seems probable that the last entry completing the work (one of the two $1+2+3+4+4^{inv}$ combinations by all means) comes above a tonic pedal point, i.e. in G minor. Of the two six-voice strettis the transposition of the subsequent one is more fortunate. (The other would be too high.)

Similarly, it may be assumed that the penultimate stretto (or the other six-voice one) appears above a dominant pedal point, thus in D minor. This transposition is also advantageous.

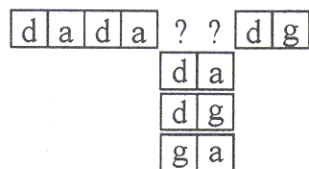
In a comparison of the form design of modulation, the permutational matrix with the keys of the second half (two entries) of the subsequent section allows the prediction of the first half as well. In order to avoid repetitions, the missing two entries can namely have three kinds of key only. Any other sequence would result in identical keys in the neighbouring positions (e.g. $d \ a \ d \ \underline{a} \ a \ g \ d$ or $d \ a \ d \ a \ g \ \underline{d} \ d \ g$).



Example 64

In this phase of the reconstruction process a peculiar modulation concept seems to get outlined almost "automatically": it is as if the key scheme of the section following the permutational matrix would be an exact reflexion of that of the matrix. The first and third entries of both the permutational matrix and the inversion matrix are in D minor, while the second and fourth entries of the permutational matrix sound in the key of the upper fifth (A minor) and the second and fourth entries of the inversion matrix in the key of the lower fifth (G minor).

Example 65



Example 66

permutational matrix	+1		a	a
	0	d	d
inversion matrix	-1		g	g

Example 67

Of the two pitch exchanges of the four-voice stretto (1+2+3+4^{inv}), the one with D minor transposition seems to be more fortunate in the following:

Example 68

In the case of a transposition to G minor the other pitch exchange gets into a favourable position.

On comparing the above sequence of the pitch exchanges with the permutations of the matrix, it can be seen that the inverted main subject enters not only in the keys reflected to the D minor axis (the note F), but also in the exactly contrary (inverted) sequence of parts, as if it were a mirror fugue.

Example 69

permutational matrix

4			
			4
		4	
	4		

d a d a

inversion matrix

4'			
		4'	
			4'
4'			

d g d g

Example 70

In this instance we cannot speak of a mirror fugue in the sense as had occurred in *Contrapuncti* 12 and 13 preceding the fugue, since the other voices are not and cannot be inverted. Nevertheless, it deserves attention that the principle of inversion is strictly observed here as well. This portion of the quadruple fugue absorbs and words anew the characteristic features of the

mirror fugues in a more complex structure, a new context and under new circumstances.

The material combining the permutational matrix with the subsequent passage (the inversion matrix) could essentially be inserted after the reconstruction of these two large formal blocks, at least of the beginning of the mirror matrix, as in the case of the measures connecting the third and fourth sections.

The authenticity of the sequence of pitch exchanges, tonal arrangement and strettos of the subject entries is guaranteed by the realization of the very restricted possibilities of the various constructional aspects. With the episodes the situation is different. Though they cannot be randomly woven, the case being that they are governed by very strict stylistic restrictions and compositional rules both with regard to melody and motifs and harmony, it is more difficult to corroborate their authenticity than that of the subject entries.

In the present reconstruction the permutational matrix and the subsequent formal block (the inversion matrix) are joined by the episode which followed the exposition of the second subject earlier (mm. 142–147). Its use was not suggested by intuition alone but also by certain considerations of form: (1) The permutational matrix agrees with the length of the exposition of the second subject (for the same reason because the second subject is the longest of the four), thus with regard to its proportions the matrix as a return can be heard mostly as the reprise of the second subject; (2) After the last entry of the matrix, similar to the exposition of the second section, a modulation needs to be carried out from A minor to the D minor of the opening entry of the subsequent block; (3) In the pitch arrangement of both target entries the second subject can be heard in the soprano, increasing thereby the similarity of the two sections. (NB The inverted main subject stated in the bass of the first entry of the inversion matrix and the first subject announced also in the bass in the fifth entry of the second section outline each others' melody.)⁵⁹ In sum: in addition to performing an important modulation task the use of the episode stresses the reprise character of the form.

It is worth comparing the last two measures that conclude the reconstruction with the coda of some other fugues. The similarity is all but by chance: inversion practices play an important role with all four fugues. Both *Contrapuncti* 5 and 6 are counter-fugues and the inversion of the subject appears several times in the strettos of the D minor Fugue of volume 1 of *The Well-Tempered Clavier* (BWV 851:2). The decisive importance of inversion in the final *Contrapunctus* does not need to be stressed. Each of the examples sums up and closes the movement in the densest form possible (with simultaneous inversion in at least two parts each).

⁵⁹ Cp. ex. 45.

Example 71

The succession and modulation sequence of the pitch exchange of the above counterpoint combinations provide the most cogent evidence – even if it seems highly hypothetical – that they "function" exclusively thus. At the same time a significant wealth of inner structural relationships unfolds that has been inconceivable so far. On the other hand, this structure opens possibilities – proving thus its validity to a non-negligible degree – for construing Bach's apocryphal oral testimony, the much-disputed Mizler's obituary relating to the final fugue. It has always been obvious that the obituary (may) contain elements of truth expressed in misunderstandable terms, in a misleading manner. (See note 38.) The "penultimate" fugue is, as a matter of fact, the third section while the "last" is the fourth, i.e. the permutational matrix and the subsequent inversion matrix. It is striking how precisely, "note for note" the inversion practices are described in the text and how important it is to stress that the inversion takes place "in all four voices", whereas it is left open *what is being*

inverted in reality – the entire section (i.e. its first part) or the four subjects – and whether the four voices are inverted simultaneously or in succession. The inversion criteria listed in the obituary ("in all four voices, note for note") can exclusively refer to the fourth subject, the main subject of the cycle as the above contrapuntal investigations have revealed and not to all four subjects, and not simultaneously to all four voices.

In the light of the earlier discussed structure Mizler's Obituary Notice of J. S. Bach in 1754 can be stated precisely: "His final illness prevented him, according to his draft, from bringing to a complete end the penultimate fugue [third section] and from working out the last one [fourth section], which was to contain four subjects and [of which the fourth subject was] to have been afterwards inverted note for note in all four voices."

5. Analysis of the Reconstructed Fugue

5.1. The Relationships of the Stretti

It follows from the principle of contrapuntal intensification that the reconstructed part reaches very soon after m. 239 the extreme complexity that manifests itself in the thematic saturation of the four obbligato voices.⁶⁰ It deserves attention that a thematically free voice can be found in the first of the reconstructed twelve entries only, and, resulting from the strict contrapuntal construction, even its play is considerably determined.

2	1	3	3	4	2	3	1	2	4 ¹	3	2
	3	1	2	2	3	1	4	3	3	4 ¹	4
1	3	2	1	3	1	4	2	1	2	1	3
3	2	3	1	1	4	2	3	4 ¹	1	2	4 ¹
										4	1

Example 72

In the final *Contrapunctus* several novelties come true regarding the contrapuntal solutions of the whole cycle, moreover, the whole oeuvre: (1) The large form itself is, of course, unique since it is the only quadruple fugue of the entire oeuvre which, as a consummation of the life-work, postulates complex

⁶⁰ The further increase of thematic saturation is naturally performed by the entry of additional, supernumerary voices.

contrapuntal structures from the beginning; (2) The invertible counterpoint at twelfth playing a leading role in *Contrapunctus* 9 (double fugue) figures in the fourth section in a very complex manner, on the scale of the four thematic voices (1+2+3+4¹²: mm. 284–289); (3) Examples of the elaboration of the subjects and counter-voices in the original position and in inversion could be seen in the earlier *Contrapuncti* as well (such a relationship prevails between the subjects of *Contrapuncti* 8 and 11⁶¹ and naturally between the rectus and inversus versions of the mirror fugues as well). On such occasions, however, all subjects and counter-subjects were inverted. In contrast, in *Contrapunctus* 14 the first three subjects in original position can be combined both with the original and inverted forms of the main subject as a kind of contrapuntal novelty, moreover, with both of them simultaneously (1+2+3+4; 1+2+3+4^{inv}; 1+2+3+4+4^{inv}); (4) In the permutational matrix a special web appears since no permutational construction has been extant in the cycle yet;⁶² (5) In connection with the above structure there is also a perfectly unique solution found in the "faithful to parameter" stretto of the entire exposition of the first three subjects.

In the example below the relationship of the stretti of the third and fourth sections can be seen in an easily surveyable form (1+2+3, 1+2+3+3, 1+1+2+3, 1+2+3+4, 1+2+3+4^{inv}, 1+2+3+4+4^{inv}) showing how the subjects of various pitch and time displacement substitute each other. (For the sake of easier comparison the subjects are given in identical key and position, as in exx. 9 and 15.)

It is to be noted that the steadiest element of the seven stretti are the second and third subjects: they occur in all combinations in an identical position throughout. Two forms of the main subject of the cycle (4, 4^{inv}) enter at the same time, inverted on the third degree of the key, while the position in time and space of the first subject changes several times, similar to the third subject of the combination 1+2+3+3, uttered per arsin et thesin.

61 For the sake of accuracy it should be noted that the second subject of *Contrapunctus* 8 and the third subject of *Contrapunctus* 11 are not the exact inversions but the retrograde inversions of each other.

62 The permutational structure is extremely rare in Bach's instrumental work. Cp. Christoph Wolff, "Die Architektur von Bachs Passacaglia", *Acta organologica* 3, (1969), pp. 190–192.
Cp. Tovey 57.

The image displays two systems of musical notation. The top system consists of seven staves, each with a treble clef and a key signature of one flat. The notation includes various rhythmic values and accidentals. The bottom system also consists of seven staves, with similar notation. To the right of the bottom system, there is a diagrammatic representation of the stretto structure, showing vertical lines and arrows that indicate the relationships and displacements between the subjects.

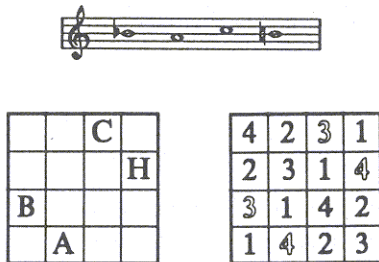
Example 73

5.2. Numerology

Reflexions received on earlier publications or papers read on the subject matter warn us that readers and listeners attribute a particular emphasis to any kind of hints at numerology, even if their importance can only be considered as secondary for the purposes of reconstruction. It is essential to bring it home because it can be rendered verisimilar that Bach conceived certain stretti – first of all the permutational matrix – by their musical characteristics in the first place and not by the possibility of their being understood through numerology. By putting figures in the place of the fairly complicated musical structures analyzed they become more lucid, easier to survey. Having the well-ordered, regular image of the figures in front of our eyes we must not forget

that their validity lies in the musical relations. On the other hand, the question arises how purposeful and conscious the phenomena are which can be interpreted in the light of numerology and whether the occurrence of some of them should not be attributed to mere chance.

As mentioned before, the "key number" in *The Art of Fugue* is 14. This figure which is the numerical equivalent of the letters of the composer's name crops up with embarrassing frequency in the most unexpected situations: (1) Bach's oeuvre abounds in examples showing what an immense significance he attributed to numbers.⁶³ It speaks for itself that prior to joining the Mizler Society (*Sozietät der musikalischen Wissenschaften*) Bach had waited for years to be its 14th member. (As a matter of fact, he wanted to present the cycle to this body as his latest work.); (2) It is also significant that *The Art of Fugue* consists of 14 *Contrapuncti* and 4 canons;⁶⁴ (3) In the last *Contrapunctus*, No. 14 the composer's name appears manifest, as the first four notes of the third subject (the sum of the alphabetical serial numbers of the letters B-A-C-H [Eng. = B-flat-A-C-B] being $2+1+3+8=14$);⁶⁵ (4) Contrapuntal reasons prove that in the final fugue the 14-note variant of the main subject of the cycle was planned;⁶⁶ (5) The "basic stretto" of the first subject occurring in the first, second and third sections alike (see ex. 3) (1+1, 1+1+2, 1+1+2+3) consists equally of 14 notes ($7+7=14$); (6) The sum of the serial numbers of the subjects extant in the last two stretti of the work is 14 as well ($1+2+3+4+4^{inv}$); (7) If the arrangement of the notes B-A-C-H (B-flat-A-C-B) according to pitch is projected to the permutational matrix, we can see that the sum of the serial numbers of the subjects occurring on the cross-shaped formis: $3+4+3+4=14$.



Example 74

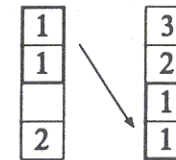
- 63 The most relevant example is the chorale setting *Vor deinen Thron tret' ich* BWV 668.
 64 NB. The earlier version of the cycle contained also 14 items (12 fugues and 2 canons).
 65 These four notes have already been mentioned as the third subject of *Contrapunctus* 11, though there it is heard on the notes E-flat-D-F-E (mm. 89–90).
 66 NB. The second subject consists of 41 notes, which is the sum of the serial numbers of the letters J. S. BACH.

The permutational matrix suggests another interpretation of numerology as well. The sum of the serial numbers of the four subjects available in each column (horizontal and vertical) of the "magic square" is $1+2+3+4=10$, which often symbolizes the Ten Commandments in Bach's oeuvre. This correlates with the strictly canonic construction of the permutational matrix: the canon (rule, standard) symbolizes imitation and obedience by virtue of voices imitating each other.⁶⁷ At the same time, the permutational rotation hides in connection with this system of symbols the principle of eternity "wie ein zufälliger Ausschnitt aus einer ewig klingenden Musik" (including the possibility of canon perpetuus as well).⁶⁸

5.3. Symmetries

In the chapter discussing the likely size of the fugue, I have already warned of the dangers lying in assuming and determining in advance the symmetries of size. In connection with the phenomena of symmetry of the reconstructed fugue listed in the following it is important to emphasize that they have not been sought for during the course of reconstruction and that they were discovered as such as by-products, "by chance". It is, of course, out of place to speak of chance with such highly organized musical material. Much rather we should speak of some kind of specific organization or system which the musical material displays from almost every aspect.

The last two stretti of the second and third sections constitute a pair in a symmetrical arrangement (1+1+2: mm. 180–188, 1+1+2+3: mm. 269–276) inasmuch as both contain two overlapping first subjects (1+1). But while this form appears at the end of the second section in the two upper voices, it does so in the two lower voices at the end of the third section, supplementing the preceding one.



Example 75

- 67 Cp. e.g. *Clavier-Übung III: "Dies sind die heil'gen zehn Gebot"* BWV 678 and the final chorus of the cantata *Du sollst Gott, deinen Herren, lieben* BWV 77.
 68 Neumann 50, p. 48.

In the relationship of the permutational matrix and inversion matrix, a much more complicated achievement of the inversion symmetry can be seen. The two forms of the main subject of the cycle (4, 4^{inv}) join these sections in a way as if they were the rectus and inversus forms of a mirror fugue (disregarding the temporal proportion and the relationship with the other parts). (See Ex. 70) In this context it can be seen that the final *Contrapunctus* has, when put into the "evolution" process of the entire cycle, a specific relationship with each of the earlier blocks raising and developing various contrapuntal practices, and sums up, so to say, the cycle by containing their essential attributes.

Fugue type

Contrapunctus 14

fugues with 1, 2 and 3 subjects (1-4, 9-10, 8, 11)	→	fugue with 4 subjects
stretto counter-fugues (5-7)	→	main subjects in two forms (4, 4 ^{inv})
mirror fugues (12-13)	→	permutational matrix – inversion matrix
4 canons	→	permutational construction

Example 76

A symmetry of a different kind comes into being by the peculiar role the first subject plays: (1) The first subject is, as mentioned at the analysis of the first section, of axial symmetrical construction. Consequently, its initial note agrees with its last note (see ex. 1); (2) The dramaturgy of the first section becomes somewhat unusual in that after the two combinations that join three subjects in stretto (1+1^{var}+1, 1+1^{inv}+1: mm. 89–104), the first subject appears alone in the last entry – not following the principle of contrapuntal intensification –, but notably in the same voice (bass), key (D minor) and position (small octave) as at the beginning of the work. As a result, the form of the first section repeats the symmetry of the first subject on a larger scale: its beginning and end are identical; (3) The first subject appears at the first stretto of the second, third and fourth sections (1+2, 1+2+3, 1+2+3+4) in the position it was stated at the beginning of the work and also in the last entry closing the movement (1+2+3+4+4^{inv}). Ex. 77 shows excellently how the contrapuntal combinations getting more and more complex and increasingly saturated thematically lie on the pillars of the first subject appearing unaltered in the bass.

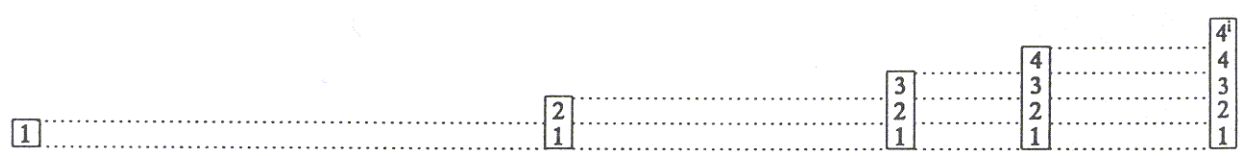
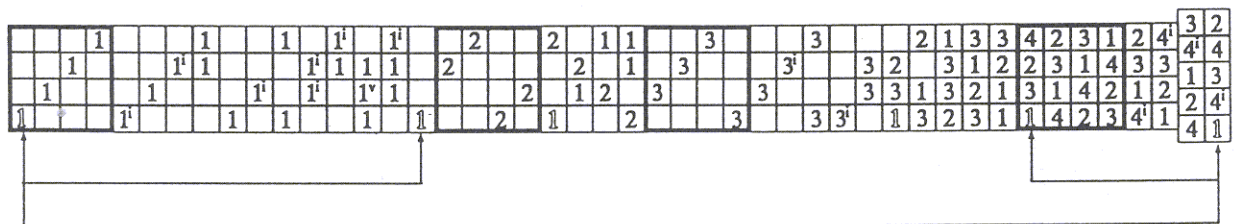
As the combination part of the fourth section coincides with the exposition of the fourth subject (permutational matrix), the bass voice of the first and last entries of the fourth section agree in the same way as they did in the first section. In this way the symmetrical trait of the first section is repeated in the dimensions of the whole quadruple fugue: the bass voice of the first and last entries is identical. The complicated structure of the following sentence is meant

to express adequately the complexity of the symmetry multiplying each other many times: The *first* and *last* notes of the first subject appearing in the *first* and *last* entries of the *first* and *last* sections of the work agree.

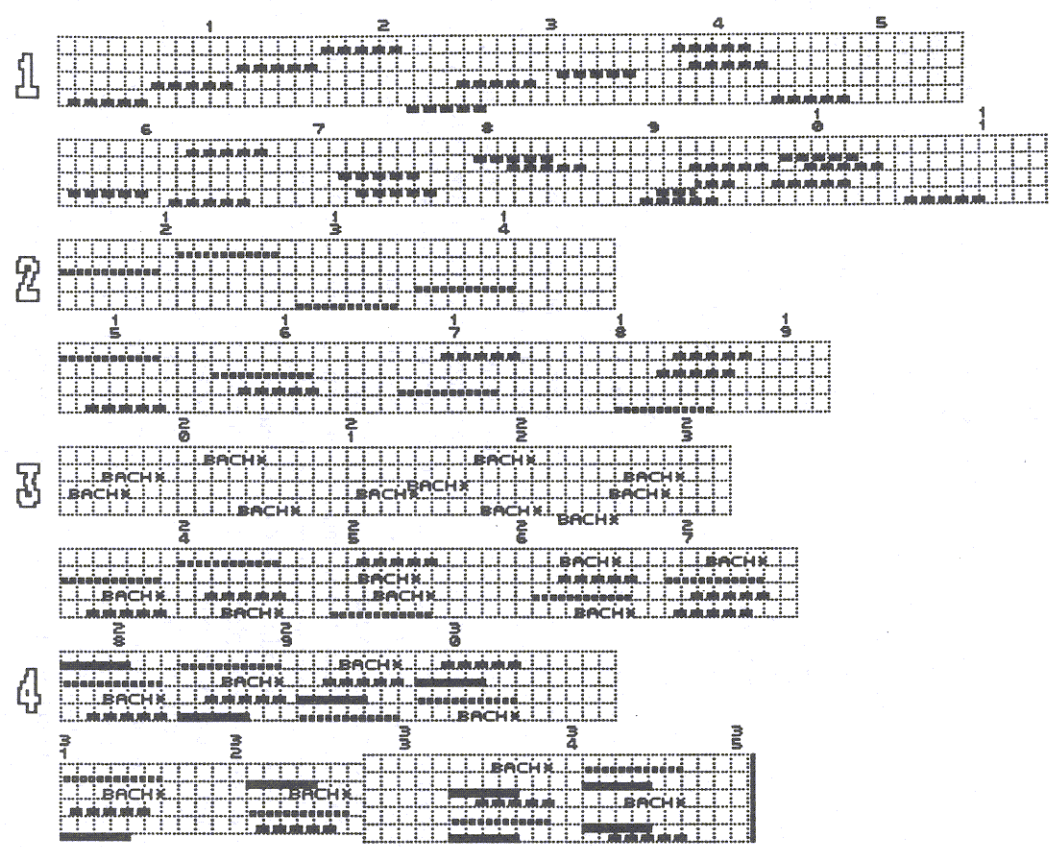
The symmetry inherent "on the level of atoms" in the subject that starts the fugue extends its effect over ever larger units of the movement until it embraces the complete universe of the composition from beginning to end. The beginning and end, the alpha and omega merge to one.

5.4. Graphic Charts

Example 77



Example 78



Summary

J. S. Bach's fugues – and this is pertinent to the missing parts of *Contrapunctus* 14 as well – come about as the synthesis of contrapuntal possibilities and constructional rules, as their optimum coincidence. The Bachian fugue itself is nothing else but the many-sided and at the same time economic, concentrated elaboration of the contrapuntal combinations of the subjects and counter-subjects. The too-lengthy development of the theoretically innumerable contrapuntal combinations is limited by the principle of shunning repetitions, that is eliminating redundancy (modulations, interchanges of voices) as well as by certain restrictions of playing technique (spacing) and harmonic, part-writing prohibitions.

In the case of *Contrapunctus* 14 the possibilities are so restricted that the development and consummation of the musical material can take place in almost one way only and the process evolves almost automatically, "on its own". This is true not only of the subject entries but of the episodes as well.

The comparison of the various strettos makes evident that the four subjects are melodic variants of each other: strung onto the chain of the second subject, the individual subjects (1, 3, 4, 4^{inv}) are capable of replacing each other in an adequate position which is due to their melodic similarities. To word it differently: all three first subjects are variants of the main subject of the cycle, its positive and negative copies, respectively.

In Bach's fugue the boundaries of form are not sharply drawn. The elements of structure span them, strengthening thereby the organic unity of the movements. In addition, in *Contrapunctus* 14, the various passages explicitly overlap in several places, combining the attributes of several sections of form which results in an extremely dense and intensive construction.

Considering the final *Contrapunctus* in the context of the whole cycle, it turns out that it contains almost all contrapuntal procedures applied in the earlier movements and thus sums up the cycle in a very concentrated form.

In the final fugue several novelties of form and counterpoint come into play, of which the permutational matrix is undoubtedly the most striking one. It is "programmed" into the exposition of the first three subjects and emerges through the stretto of these blocks and lends the cycle an encyclopedic wealth.

Resulting from the completion, there unfold a lot of relationships to be understood numerologically and several phenomena of symmetry of both time and space.

The highly organized musical material shows an exceptional arrangement in almost every aspect. Its crystallike construction develops organically and creates time, space and universe on its own as the message of a Master bringing his life and art to consummation.

...comprendió que él también era una apariencia,
que otro estaba soñándolo.

Jorge Luis Borges: Concentric Ruins